**Read Me First:**   *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to".   Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.*

1. (15 pts.)   (a) (5 pts.) Carefully sketch both $f(x) = \ln(x)$ and $g(x) = e^x$ on the coordinate system below. **Label very carefully**.

$$y \uparrow$$

$$\longrightarrow x$$
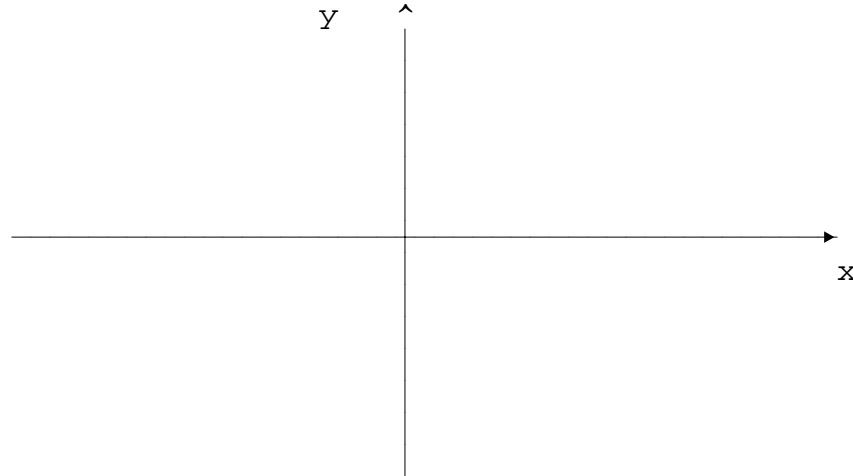
(b) (10 pts.) Evaluate each of the following limits.

$\lim\limits_{x \to \infty} \ln(x) =$ $\lim\limits_{x \to 0^+} \ln(x) =$

$\lim\limits_{x \to \infty} e^x =$ $\lim\limits_{x \to -\infty} e^x =$

$\lim\limits_{x \to \infty} \left(1 + \dfrac{1}{x}\right)^x =$

2. (10 pts.) If $f(x) = \dfrac{1}{x}$, find

$$\frac{f(x+h) - f(x)}{h} ,$$

and simplify as much as possible algebraically. Kindly observe that there are no limits of any sort being taken here.

$$\frac{f(x+h) - f(x)}{h} =$$

3. (25 pts.)   For each of the following, find the limit if the limit exists. If the limit fails to exist, say so. Be as precise as possible here. [Work on the back of Page 1 of 4 if you run out of room here.]

(a) $\lim\limits_{x \to 2} \dfrac{x^2-x-2}{x^2-1} =$

(b) $\lim\limits_{x \to -1} \dfrac{x^2-x-2}{x^2-1} =$

(c) $\lim\limits_{x \to 4} \dfrac{x^{1/2} - 2}{4 - x} =$

(d) $\lim\limits_{x \to 4^-} \dfrac{2x - 8}{|x - 4|} =$

(e) $\lim\limits_{x \to 1} (20x^2 - 4)^{1/2} =$

4. (15 pts.)   Suppose that

$$h(x) = \begin{cases} 2x - x^2 & , \ \textit{if} \ x > 1 \\ 2 & , \ \textit{if} \ x = 1 \\ \dfrac{4x^4 - 2}{3 + x^4} & , \ \textit{if} \ x < 1 \end{cases}$$

Evaluate each of the following easy limits.

(a) $\lim\limits_{x \to \infty} h(x) =$

(b) $\lim\limits_{x \to -\infty} h(x) =$

(c) $\lim\limits_{x \to 2} h(x) =$

(d) $\lim\limits_{x \to 0} h(x) =$

(e) $\lim\limits_{x \to 1} h(x) =$

5. (10 pts.)   Here are five trivial limits to evaluate:

(a) $\lim\limits_{x \to -\infty} (-4) =$

(b) $\lim\limits_{h \to \infty} (-\pi h) =$

(c) $\lim\limits_{z \to -\infty} \dfrac{|\pi z|}{2z} =$

(d) $\lim\limits_{x \to \infty} \left(\dfrac{\pi}{2} - \dfrac{3}{\ln(x)} \right) =$

(e) $\lim\limits_{x \to -\infty} e^{-x^2} =$

6. (5 pts.) Using complete sentences and appropriate notation, provide the precise mathematical definition of a **function** f //

7. (5 pts.)   Express the following function in piecewise defined form without using absolute values:

$$f(x) = |x-2| + |x|$$

8. (5 pts.)
Express $f(x) = 2 \cdot \cos^2(5x^4)$ as the composition of two functions g and h with f = g∘h, that is find g and h so that f(x) = (g∘h)(x).

g(x) =

h(x) =

Now check your work by correctly computing (g∘h)(x).

(g∘h)(x) =

9. (10 pts.)   Evaluate each of the following thorny limits:

(a) $\lim\limits_{x \to \infty} [(x^2+4x+4)^{1/2} - x] =$

(b) $\lim\limits_{x \to 0} \dfrac{(9+x)^{1/2} - 3}{x} =$

Silly 10 point Bonus Problem: Let

$$f(x) = (\ln(x) - \ln(3)) \cdot (e^x - e), \; x > 0.$$

Determine the open intervals where f(x) > 0 and the open intervals where f(x) < 0.   [Work on the back of page 3 of 4.]