
Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me all the magic on the page. Eschew obfuscation.*

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1. (25 pts.) Compute the derivatives of the following functions. You may use any of the rules of differentiation that are at your disposal. Do not attempt to simplify the algebra in your answers.

(a) $f(x) = 4x^{12} - 7x^{-6} + 8\tan(x)$

$f'(x) =$

(b) $g(x) = (4x^{1/2} - 8x^{-1/4})\sec(x)$

$g'(x) =$

(c) $h(t) = \dfrac{5\cot(t)}{12t^{2/3}}$

$h'(t) =$

(d) $y = \sin^3(\tan(2\theta + 1))$

$\dfrac{dy}{d\theta} =$

(e) $L(z) = \cos(8z^4) + 4\csc\left(\dfrac{\pi}{2}\right) - 4\csc\left(\dfrac{z}{2}\right)$

$\dfrac{dL}{dz}(z) =$

2. (10 pts.) (a) Using complete sentences and appropriate notation, provide the precise mathematical definitions for **continuity** of a function f(x) at a point x = a.

(b) Is there a real number k, that will make the function f(x) defined below continuous at x = 0 ? Either find the value for k and using the definition, prove that it makes f continuous at x = 0, or explain why there cannot be such a number k. Suppose

$$f(x) = \begin{cases} \tan(\pi x)/(4x) & , \ x \neq 0 \\ k & , \ x = 0 \end{cases}$$

3. (10 pts.) (a) Using complete sentences and appropriate notation, state the Intermediate Value Theorem.

(b) Use the Intermediate Value Theorem to prove the equation

$$\sin(x) = -\frac{3}{4}$$

has at least one real solution in the interval $[5\pi/2, 7\pi/2]$.

4. (5 pts.) Reveal all the details in showing how to use the squeezing theorem to obtain the following limit:

$$\lim_{x \to 0} x^2 \sin^2\left(\frac{1}{x}\right) =$$

[Warning: A numerical value without an explanation is worth nothing.]
Explanation:

5. (15 pts.) (a) (10 pts.) Using implicit differentiation, compute dy/dx and d^2y/dx^2 when $x^2 + y^3 = -9$. **Label your expressions correctly or else.**

(b) (5 pts.) Obtain an equation for the line tangent to the graph of $x^2 + y^3 = -9$ at the point $(-1,-(10)^{1/3})$.

6. (10 pts.) (a) Find formulas for Δy and the differential dy when $y = x^3$. Label your expressions correctly using an equal sign.

(b) Use an appropriate linear approximation formula to estimate $(101)^{1/2}$.

7. (10 pts.) (a) Using complete sentences and appropriate notation, provide the precise mathematical definition for the derivative, f′(x), of a function f(x).

(b) Using only the definition of the derivative as a limit, show all steps of the computation of f′(x) when $f(x) = 4x^{-1}$.

f′(x) =

8. (5 pts.) Pretend f is a magical function that has the property that at $x = 2\pi$ the tangent line f is actually defined by the equation $y = -4\pi(x - \pi) + 3\pi^2$. Obtain

(a) $f'(2\pi)$ = (b) $f(2\pi)$ =

9. (10 pts.) (a) Using complete sentences and appropriate notation, provide the precise mathematical definition for

$$\lim_{x \to a} f(x) = L.$$

(b) Using only the mathematical definition of limit, provide a complete proof that

$$\lim_{x \to -1} (7x + 5) = -2.$$

Silly 10 point Bonus Problem: Prove that if sine and cosine have a derivative at one point x_0, then sine and cosine have a derivative at every real number x and write sin′(x) and cos′(x) in terms of $\sin'(x_0)$ and $\cos'(x_0)$. [Say where your work is. It won't fit here. **Hint:** Trig or treat derivative definition.]