
Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me all the magic on the page. Eschew obfuscation.*

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1. (10 pts.) Determine the maximum and minimum values of the function $f(x) = x^3 - 2x^2$ on the interval [1,4] and where they occur. Before you attempt to find them, explain how you know f(x) actually has both a maximum and a minimum. [Hint: Pay attention to the domain. Watch out for "critical points" that aren't.]

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2. (10 pts.) (a) Use logarithmic differentiation to find dy/dx when

$$y = (x^4 + 1)^{\sec^{-1}(x)}.$$

(b) Find dy/dx by using implicit differentiation when

$$\tan^{-1}(xy) = \tan(x + y).$$

3. (10 pts.) (a) State the Mean Value Theorem of Differential Calculus. Use a complete sentence and appropriate notation.

(b) By using the Mean Value Theorem on the interval [3,4] with respect to the function $f(x) = x^{1/2}$, show that $1.71 < 3^{1/2} < 1.75$.

4. (10 pts.) Evaluate each of the following limits. If a limit fails to exist, say how as specifically as possible.

(a) $\lim_{x \to 0} \frac{\sin^{-1}(3x)}{x} =$

(b) $\lim_{x \to +\infty} \left(1 - \frac{3}{x}\right)^{x} =$

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5. (10 pts.) A rectangular area of 1600 square feet is to be fenced. Three of the sides will use fencing costing $4.00 per running foot, and the remaining side will use fencing costing $1.00 per running foot. Find the dimensions of the rectangle which lead to the least cost to fence the area, and prove that these dimensions actually result in the minimum cost by means of an appropriate analysis.

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6. (5 pts.) Rolle's Theorem states that if f(x) is continuous on [a,b] with f(a) = f(b) = 0 and differentiable on (a,b), then there is a number c in (a,b) such that f′(c) = 0. **Find an example of a function f(x) defined on [-1,1] with f being differentiable on (-1,1) with f(-1) = f(1) = 0 but such that there is no number c in (-1,1) with f′(c) = 0.** [Hint: Which hypothesis above must you violate??]

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7. (5 pts.) Find the function f(x) which satisfies the following two equations: $f'(x) = 43x^{42} + 5x^4$ for all x and f(-1) = 4.

8. (5 pts.) Do you really understand equations with integral signs?? Find the function g(x) that satisfies the following equation

$$\int g(x)\,dx = \sin(x) - x\cos(x) + C$$

g(x) =

9. (15 pts.) Evaluate each of the following integrals or anti-derivatives.

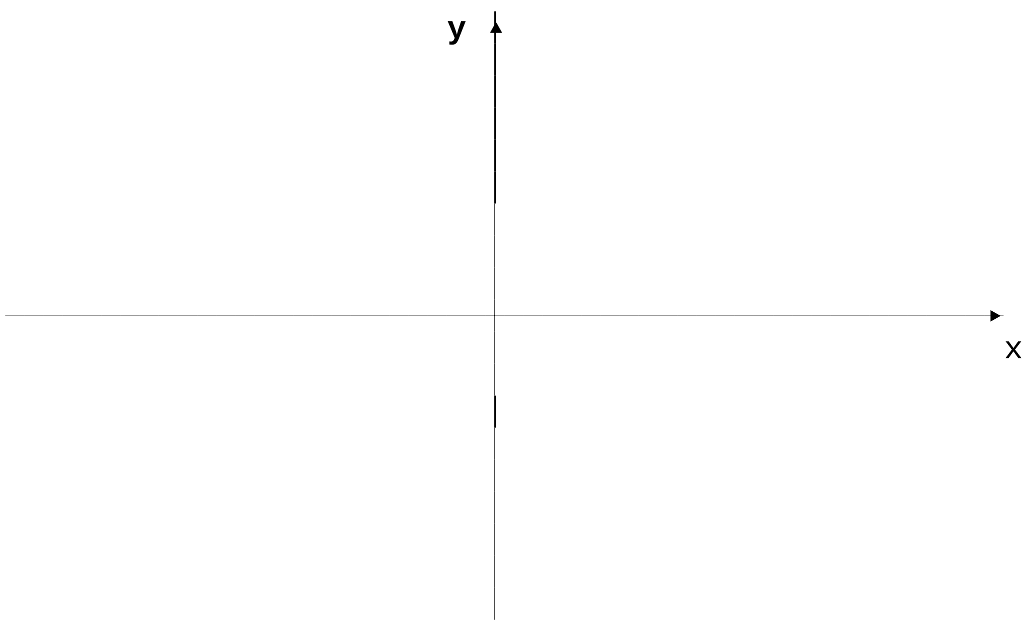
(a) $\displaystyle\int 3x^2 + \frac{1}{x} + 3\sec^2(x)\,dx =$

(b) $\displaystyle\int \frac{2x^3 + 4x^2 + 1}{x^2}\,dx =$

(c) $\displaystyle\int \frac{1}{1 + 4x^2} + (4x + 2)e^{x^2 + x}\,dx =$

10. (20 pts.) Fill in the blanks of the following analysis with the correct terminology. Then sketch the graph of the function analyzed on the coordinate system provided. Label carefully.

Let $f(x) = 2x^3 - 6x^2 + 4$. Then $f'(x) = 6x^2 - 12x$ $= 6(x - 0)(x - 2)$. Consequently, $x = 0$ and $x = 2$ are ____________________ points of f. Since $f'(x) > 0$ for $x < 0$ or $x > 2$, f is ____________________ on the set $(-\infty,0)\cup(2,\infty)$. Also, because $f'(x) < 0$ when $0 < x < 2$, f is _______________________ on the interval (0,2). Using the first derivative test, it follows that f has a(n) ______________________________________ at $x = 0$, and a(n) _______________________________________ at $x = 2$. Since $f''(x) = 12(x - 1)$, we have $f''(1) = 0$, $f''(x) < 0$ when $x < 1$, and $f''(x) > 0$ when $x > 1$. Thus, f is ___________________________ on the interval $(-\infty,1)$, f is _________________________ on $(1,\infty)$, and f has a(n) _____________________________ at $x = 1$.



Silly 10 point Bonus Problem: Show how to use the Mean Value Theorem to prove the following result: If f is continuous on a closed interval [a,b] with $f'(x) < 0$ for each x in (a,b), then f is decreasing on the closed interval [a,b]. [Say where your work is, for it won't fit here!]