Student Number: **Exam Number:**

Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me all the magic on the page. Eschew obfuscation.*

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1. (30 pts.) Very basic derivatives .... Provide the first derivative for each of the following functions. 2 points/part.

(a) $f(x) = \log_{50}(x)$ $f'(x) =$

(b) $f(x) = 50^x$ $f'(x) =$

(c) $f(x) = \tan^{-1}(x)$ $f'(x) =$

(d) $f(x) = e^x$ $f'(x) =$

(e) $f(x) = \sin(x)$ $f'(x) =$

(f) $f(x) = \cos^{-1}(0)$ $f'(x) =$

(g) $f(x) = \tan(x)$ $f'(x) =$

(h) $f(x) = \sec(x)$ $f'(x) =$

(i) $f(x) = \csc(x)$ $f'(x) =$

(j) $f(x) = \cot(x)$ $f'(x) =$

(k) $f(x) = \sin^{-1}(x)$ $f'(x) =$

(l) $f(x) = \ln(x)$ $f'(x) =$

(m) $f(x) = \sec^{-1}(x)$ $f'(x) =$

(n) $f(x) = x^{50}$ $f'(x) =$

(o) $f(x) = \cos(x)$ $f'(x) =$

2. (30 pts.) Provide each of the following antiderivatives. **Do not forget the arbitrary constant.** 2 points/part.

(a) $\int e^x \, dx$ =

(b) $\int x^{23} \, dx$ =

(c) $\int \sin(x) \, dx$ =

(d) $\int x^{-1} \, dx$ =

(e) $\displaystyle\int \frac{1}{(1 - x^2)^{1/2}} \, dx$ =

(f) $\int \sec(x) \cdot \tan(x) \, dx$ =

(g) $\displaystyle\int \frac{1}{|x|(x^2 - 1)^{1/2}} \, dx$ =

(h) $\int \sec^2(x) \, dx$ =

(i) $\int \cos(x) \, dx$ =

(j) $\displaystyle\int \frac{1}{1 + x^2} \, dx$ =

(k) $\int 23^x \, dx$ =

(l) $\int \csc^2(x) \, dx$ =

(m) $\int 0 \, dx$ =

(n) $\int \csc(x) \cdot \cot(x) \, dx$ =

(o) $\int 50^{50} \, dx$ =

3. (15 pts.) Evaluate each of the following easy limits. If a limit fails to exit, say how as precisely as possible. 1 point/part.

(a) $\lim\limits_{x \to \infty} e^x =$

(b) $\lim\limits_{x \to 0^-} \dfrac{1}{x} =$

(c) $\lim\limits_{x \to -\infty} e^x =$

(d) $\lim\limits_{x \to \pi/2} \dfrac{\sin(x)}{x} =$

(e) $\lim\limits_{x \to \infty} \ln\left(1 + \dfrac{1}{x}\right) =$

(f) $\lim\limits_{x \to \infty} \dfrac{\sin(x)}{x} =$

(g) $\lim\limits_{x \to \infty} \left(1 + \dfrac{1}{x}\right)^x =$

(h) $\lim\limits_{x \to \frac{1}{\sqrt{3}}} \tan^{-1}(x) =$

(i) $\lim\limits_{x \to 0} e^x =$

(j) $\lim\limits_{x \to -\infty} \tan^{-1}(x) =$

(k) $\lim\limits_{x \to 0} \dfrac{\sin(x)}{x} =$

(l) $\lim\limits_{x \to 0^+} \ln(x) =$

(m) $\lim\limits_{x \to -\infty} \left(2 + \dfrac{1}{x}\right)\left(\dfrac{2}{x} - 3\right) =$

(n) $\lim\limits_{x \to 0} \dfrac{\cos(\frac{\pi}{3} + x) - \cos(\frac{\pi}{3})}{x} =$

(o) $\lim\limits_{x \to 2} \left(1 + \dfrac{1}{x}\right)^x =$

4. (20 pts.) Compute the derivatives of the following functions. You may use any of the rules of differentiation that are at your disposal. Do not attempt to simplify the algebra in your answers. 5 pts./part

(a) $f(x) = \tan(3x^2)\ln(\pi x)$

$f'(x) =$

(b) $g(x) = \dfrac{e^{4x}}{x^2+1}$

$g'(x) =$

(c) $h(t) = \sin(\cos^{-1}(3x^2))$

$h'(t) =$

(d) $y = 35\sin(\pi\theta^2) - \dfrac{5}{\theta^{10}} + \ln(1)$

$\dfrac{dy}{d\theta} =$

5. (10 pts.)

Find d^2y/dx^2 by implicit differentiation when $y + \sin(y) = x$.

6. (5 pts.) Find a formula for the function g(x) which satisfies the following initial value problem:

$$g'(x) = 4\cos(x) + 2x \;\; ; \;\; g(\frac{\pi}{2}) = 0.$$

7. (5 pts.) Use logarithmic differentiation to differentiate

$$f(x) = \frac{x^{\ln(x)}}{e^{x}}$$

for $x \in (0, \infty)$.

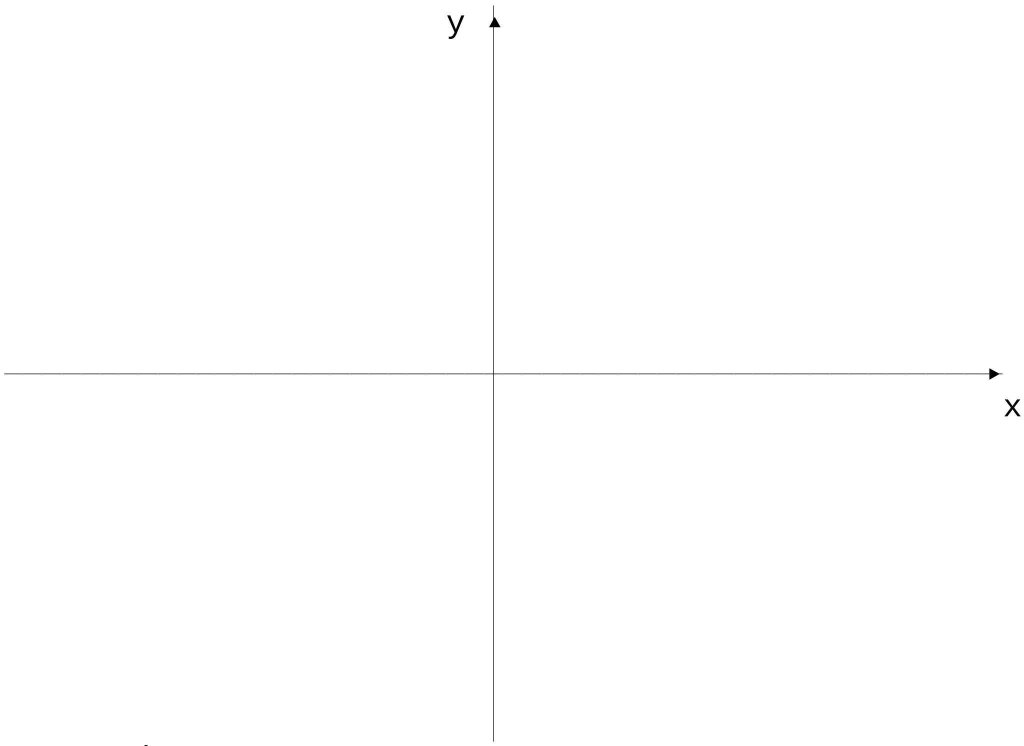
8. (5 pts.) Using only the definition of limit at a point in terms of epsilons and deltas, give a proof that

$$\lim_{x \to 2} \frac{x^2 - 4}{2x - 4} = 2.$$

Hint: This is really linear, really.

9. (10 pts.) A closed rectangular container with a square base is to have a volume of 16 cm^3. It costs twice as much per square centimeter for the top and bottom as it does for the sides. Find the dimensions of the container with the least cost and prove the cost is a minimum.

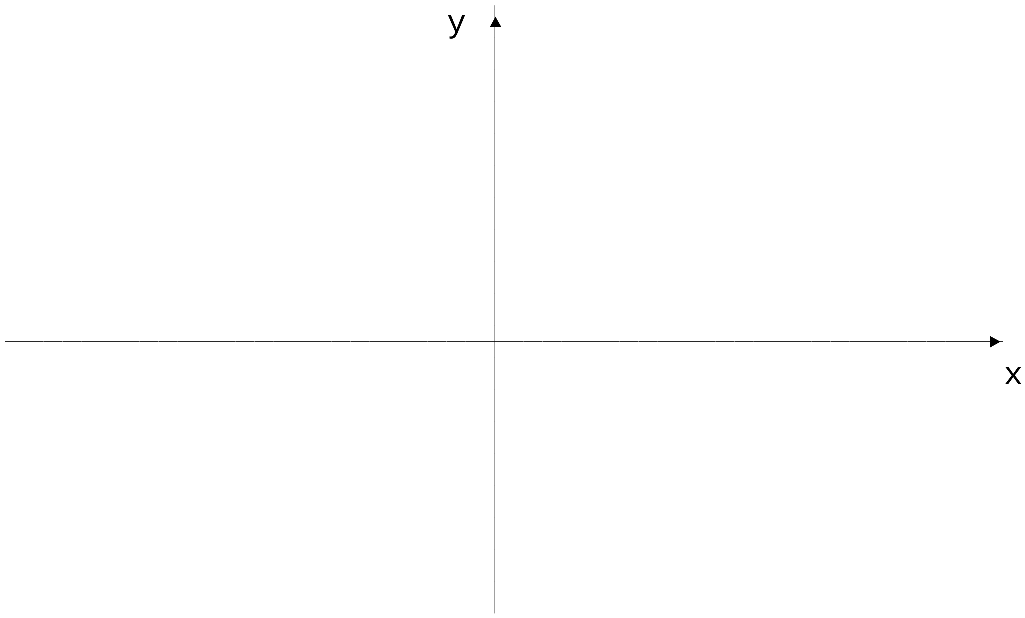
10. (15 pts.)
Let $f(x) = 4x^3 - 3x^4$. Analyze f' and f'' and how f behaves as $x \to \pm\infty$. Plot critical points, inflection points, and zeros. Then sketch the graph carefully below:



Analysis (10 pts.):

11. (5 pts.) The side of a large cube is measured to be 10 ft, with a possible error of ±0.1 ft. Using differentials, estimate the percent error in the calculated volume of the cube.

12. (15 pts.) (a) Sketch the curve defined by the parametric equations $x = 1 + \cos(t)$ and $y = 1 - \sin(t)$ with $0 \leq t \leq 2\pi$ by eliminating the parameter t, and indicate the direction of increasing t.



(b) Write an equation for the line tangent to this curve at the point on the curve where $t_0 = 3\pi/4$. **Hint:** It helps to compute dy/dx at $t_0 = 3\pi/4$ without eliminating the parameter.

(c) Compute d^2y/dx^2 at $t_0 = 3\pi/4$ without eliminating the parameter.

13. (5 pts.) A spherical weather balloon is inflated so that its volume is increasing at a constant rate of 4 cubic feet per minute. How fast is the diameter of the balloon changing when the radius is 7 ft.?
[In case you forgot, $V = (4/3)\pi r^3$.]

14. (10 pts.) Locate and determine the maximum and minimum values of the function $f(x) = 3x^2 - x^3$ on the interval [-1, 1]. What magic theorem allows you to conclude that f(x) has a maximum and minimum even before you attempt to locate them? Why??

15. (10 pts.) Evaluate each of the following limits. L'Hopital may help. Squeezing may help. Nothing may help.

(a) $\lim\limits_{x \to 0} \dfrac{8 - 8\cos(2x)}{e^x + e^{-x} - 2} =$

(b) $\lim\limits_{t \to \infty} 30t \sin(\dfrac{5}{t}) =$

16. (10 pts.) Evaluate each of the following antiderivatives.

(a) $\int x e^{2x^2+1} + \dfrac{\ln^3(x)}{x}\, dx =$

(b) $\int \dfrac{1}{(9+x^2)^{1/2}} + \dfrac{5x}{1+x^4} dx =$

10 Bonkers Bonus Points: For each of the following multiple choice or true/false questions, circle the single most appropriate answer. Work on the back of page 8.

(i) If $f(x) = e^{2x}$, then the n^{th} derivative of f at x = 1 is given by

(a) $f^{(n)}(1) = 2^2e^n$ (b) $f^{(n)}(1) = 2^ne^2$ (c) $f^{(n)}(1) = 2^ne^n$

(d) $f^{(n)}(1) = 2^n$ (e) None of the above.

(ii) Suppose that f is continuous on the open interval (-5,0) and that the tangent line to the graph of f at $x = -2$ is given by the equation $3x + 4y = 22$. Then

(a) $f'(-2) = 3/4$.

(b) $f(-2) = 28$

(c) if x is sufficiently close to -2 and $-2 < x$, then $f(x) < 7$.

(d) if x is sufficiently close to -2 and $-2 < x$, then $f(x) > 7$.

(e) none of the above is true concerning f.

(iii) When f is defined by

$$f(x) = \frac{2x}{x^2+1}$$

on the real line, it turns out that

$$f'(x) = \frac{-2(x-1)(x+1)}{(x^2+1)^2} \quad \text{and} \quad f''(x) = \frac{4x(x-\sqrt{3})(x+\sqrt{3})}{(x^2+1)^3}.$$

Which of the following is true about f on the open interval (0,1)?

(a) f is decreasing and concave down.

(b) f is decreasing and concave up.

(c) f is increasing and concave down.

(d) f is increasing and concave up.

(e) None of the above is true about f on (0,1).

(iv) If f is differentiable on (-2,4) and f(-2) = f(4) = 0, then there is a point c is the closed interval [-2,4] where $f'(c) = 0$.

True False

(v) Suppose that

$$f(x) = \frac{x+1}{x^2-4x-5}.$$

Since f(-2) = -1/7 and f(6) = 1, the Intermediate-Value Theorem implies that there must be a number $c \in (-2,6)$ where $f(c) = 0$.

True False