
Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.*

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1. (10 pts.) Here are five trivial limits to evaluate:

(a) $\lim_{y \to -\infty} \frac{4y}{|y|} =$

(b) $\lim_{x \to \infty} \frac{1}{\ln x} =$

(c) $\lim_{z \to -\infty} \left(7 - \frac{1}{z}\right) =$

(d) $\lim_{x \to -\infty} e^{-x^2} =$

(e) $\lim_{h \to -\infty} (-11h) =$

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2. (15 pts.) Suppose that

$$h(x) = \begin{cases} 2x - x^2 & , \textit{if } x < 2 \\ 2 & , \textit{if } x = 2 \\ \dfrac{x^3 - 2}{3 + 3x^3} & , \textit{if } x > 2 \end{cases}$$

Evaluate each of the following easy limits.

(a) $\lim_{x \to \infty} h(x) =$

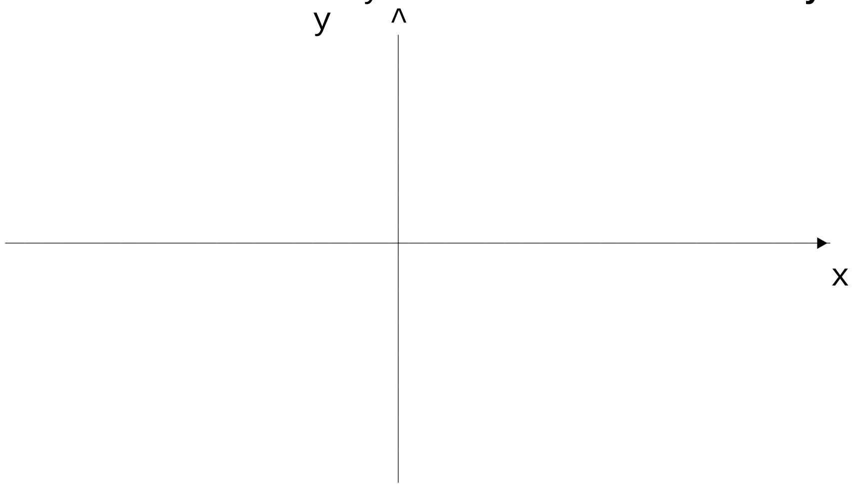
(b) $\lim_{x \to -\infty} h(x) =$

(c) $\lim_{x \to 2} h(x) =$

(d) $\lim_{x \to 0} h(x) =$

(e) $\lim_{x \to 4} h(x) =$

3. (15 pts.) (a) (5 pts.) Carefully sketch both f(x) = ln(x) and g(x) = e^x on the coordinate system below. **Label very carefully**.



(b) (10 pts.) Evaluate each of the following limits.

$\lim_{x \to \infty} \ln(x) =$

$\lim_{x \to 0^+} \ln(x) =$

$\lim_{x \to \infty} e^x =$

$\lim_{x \to -\infty} e^x =$

$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{3x} =$

4. (10 pts.) (a) Using complete sentences and appropriate notation, provide the precise mathematical definition for

$$\lim_{x \to a} f(x) = L.$$

(b) Using only the mathematical definition of limit, provide a complete proof that

$$\lim_{x \to 0} \frac{2x^2 + x}{x} = 1.$$

5. (25 pts.) For each of the following, find the limit if the limit exists. If the limit fails to exist, say so. Be as precise as possible here. [Work on the back of Page 2 of 4 if you run out of room here.]

(a) $\lim_{x \to 2} \frac{x^2-x-2}{x^2-4} =$

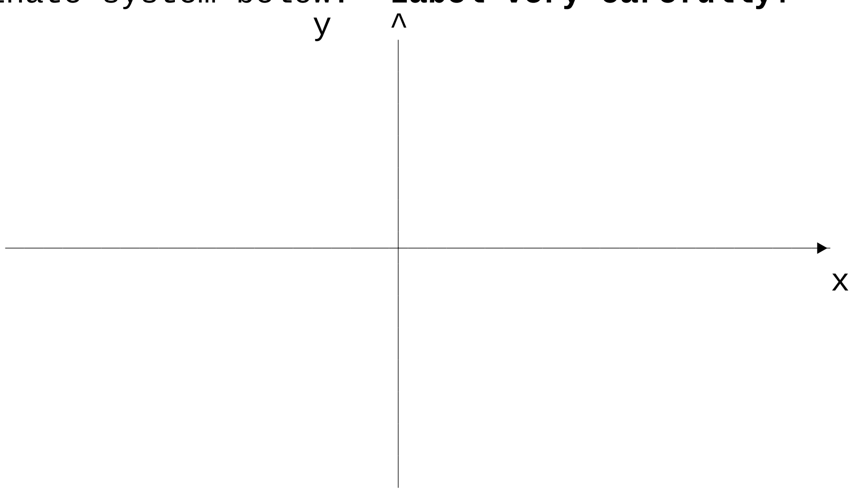
(b) $\lim_{x \to -2^+} \frac{x^2-x-2}{x^2-4} =$

(c) $\lim_{x \to 4} \frac{4 - x}{x^{1/2} - 2} =$

(d) $\lim_{x \to 5^-} \frac{3x - 15}{|x - 5|} =$

(e) $\lim_{x \to 1} (20x^2 - 4)^{1/4} =$

6. (7 pts.) Carefully sketch the function $f(x) = \tan^{-1}(x)$ on the coordinate system below. **Label very carefully**.



Then evaluate the following two limits:

$\lim_{x \to \infty} \tan^{-1}(x) =$ $\lim_{x \to -\infty} \tan^{-1}(x) =$

7. (8 pts.) If $f(x) = 3x^2 - 5$ and $h \neq 0$, then simplifying as much as possible allows us to write

$$\frac{f(x+h) - f(x)}{h} =$$

8. (10 pts.) Evaluate each of the following thorny limits:

(a) $\lim_{x \to 0} \frac{(16+x)^{1/2} - 4}{x} =$

(b) $\lim_{x \to \infty} [(x^2+3x)^{1/2} - x] =$

Silly 10 point Bonus Problem: Provide an $\varepsilon - \delta$ proof that

$$\lim_{x \to 9} x^{1/2} = 3.$$

Say where your work is, for it won't fit here.