
Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇨" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me all the magic on the page. Eschew obfuscation.*

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1. (25 pts.) Compute the derivatives of the following functions. You may use any of the rules of differentiation that are at your disposal. Do not attempt to simplify the algebra in your answers.

(a) $f(x) = 4x^6 - 7x^{-12} + 8\cos(x)$

$f'(x) =$

(b) $g(x) = (4x^4 - 8x^{-2})\sin(x)$

$g'(x) =$

(c) $h(t) = \dfrac{\sec(t)}{10t^5}$

$h'(t) =$

(d) $y = \tan(csc(2\theta + 1))$

$\dfrac{dy}{d\theta} =$

(e) $L(z) = \cot(4z^8) + 4\sec(\dfrac{\pi}{4}) - 4\sec(\dfrac{z}{2})$

$\dfrac{dL}{dz}(z) =$

2. (10 pts.) (a) Using complete sentences and appropriate notation, provide the precise mathematical definition of **continuity** of a function f(x) at a point x = a.

(b) Is there a nonzero real number k, that will make the function f(x) defined below continuous at x = 0 ? Either find the value for k and using the definition, prove that it makes f continuous at x = 0, or explain why there cannot be such a number k. Suppose

$$f(x) = \begin{cases} \tan(kx)/x\ , & x < 0 \\ 3x + 2k^2\ , & x \geq 0 \end{cases}$$

3. (10 pts.) (a) Using complete sentences and appropriate notation, state the Intermediate Value Theorem.

(b) Use the Intermediate Value Theorem to prove the equation

$$x^4 + x - 1 = 0$$

has at least one real solution in the interval [-1, 1].

4. (5 pts.) Reveal all the details in showing how to use the squeezing theorem to obtain the following limit:

$$\lim_{x \to 0} x \sin\left(\frac{1}{x}\right) =$$

[Warning: A numerical value without an explanation is worth nothing.]

Explanation:

5. (5 pts.) Compute f″(x) when

$$f(x) = \sin(x^4).$$

6. (5 pts.) Find an equation for the tangent line to the graph of

$$y = x\cos(3x)$$

when $x = \pi$.

7. (10 pts.) (a) Find formulas for Δy and the differential dy when $y = x^2 - 2x + 1$. Label your expressions correctly.

(b) Use an appropriate local linear approximation formula to estimate $(65)^{1/2}$.

8. (5 pts.) The side of the square is measured to be 10 ft, with a possible error of ±0.1 ft. Using differentials estimate the percent error in the calculated area.

9. (10 pts.) (a) Using complete sentences and appropriate notation, provide the precise mathematical definition for the derivative, $f'(x)$, of a function $f(x)$.

(b) Using only the definition of the derivative as a limit, show all steps of the computation of $f'(x)$ when $f(x) = 4x^2 + 3$.

$f'(x) =$

10. (5 pts.) Pretend f is a magical function that has the property that at x = 2 the tangent line f is actually defined by the equation y = -4(x - 1) + 3. Obtain

(a) $f'(2) =$ (b) $f(2) =$

11. (10 pts.) A softball diamond is a square whose sides are 60 feet long. Suppose a player running from first to second base has a speed of 25 feet/second at the instant when she is 10 feet from second base. At what rate is the player's distance from home plate changing at that instant?

Silly 10 point Bonus Problem: Suppose that *x is measured in degrees*. With this hypothesis, compute $\sin'(x)$ and $\cos'(x)$. Say where your work is, for it won't fit here.