Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me all the magic on the page. Eschew obfuscation.*

1. (10 pts.) Determine the maximum and minimum values of the function

$$f(x) = 10x^2 - x^4$$

on the interval [-1,5] and where they occur. Before you attempt to find them, explain how you know f(x) actually has both a maximum and a minimum.

2. (10 pts.) (a) Use logarithmic differentiation to find dy/dx when

$$y = (x^3 - 2x)^{\ln(x)}.$$

(b) Find dy/dx by using implicit differentiation when

$$x^3 + x\tan^{-1}(y) = e^y.$$

3. (10 pts.) (a) State the Mean Value Theorem of Differential Calculus. Use a complete sentence and appropriate notation.

(b) Let $f(x) = x^{2/3}$, $a = -1$ and $b = 8$. Show there is no point c in the open interval (a,b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a},$$

and explain why this does not violate the Mean-Value Theorem.

4. (10 pts.) Evaluate each of the following limits. If a limit fails to exist, say how as specifically as possible.

(a) $\lim\limits_{x \to \pi/2^-} \sec(3x)\cos(5x) =$

(b) $\lim\limits_{x \to +\infty} \left[x - \ln(1 + 2e^x) \right] =$

5. (10 pts.) A closed rectangular container with a square base is to have a volume of 2000 cm^3. It costs twice as much per square centimeter for the top and bottom as it does for the sides. Find the dimensions of the container with the least cost.

6. (5 pts.) Do you really understand equations with integral signs?? Find the function $g(x)$ that satisfies the following equation

$$\int g(x)\,dx = \sec^2(x) - \cos(x) + C$$

$g(x) =$

7. (5 pts.) Solve the following initial-value problem:

$$\frac{dy}{dx} = \frac{x+1}{\sqrt{x}}, \quad y(1) = 0.$$

8. (8 pts.) Assume f is continuous everywhere. If

$$f'(x) = x^4(e^x - 3),$$

find all the critical points of f and at each stationary point apply the second derivative test to determine relative extrema, if possible. If the second derivative test fails at a critical point, apply the first derivative test to determine the true state of affairs there.

9. (12 pts.) Evaluate each of the following integrals or anti-derivatives.

(a) $\int 3x^2 + \frac{1}{x^2+1} + 3csc^2(x)\, dx =$

(b) $\int \frac{x^3 + x + 1}{x^2}\, dx =$

(c) $\int \frac{1}{(1 - 4x^2)^{1/2}} + (4x - 2)e^{x^2 - x}\, dx =$

10. (20 pts.) When f is defined by

$$f(x) = \frac{2x}{x^2+1}$$

on the real line, it turns out that

$$f'(x) = \frac{-2(x-1)(x+1)}{(x^2+1)^2} \quad \text{and} \quad f''(x) = \frac{4x(x-\sqrt{3})(x+\sqrt{3})}{(x^2+1)^3} .$$

(a) (3 pts.) What are the critical point(s) of f and what is the value of f at each critical point?

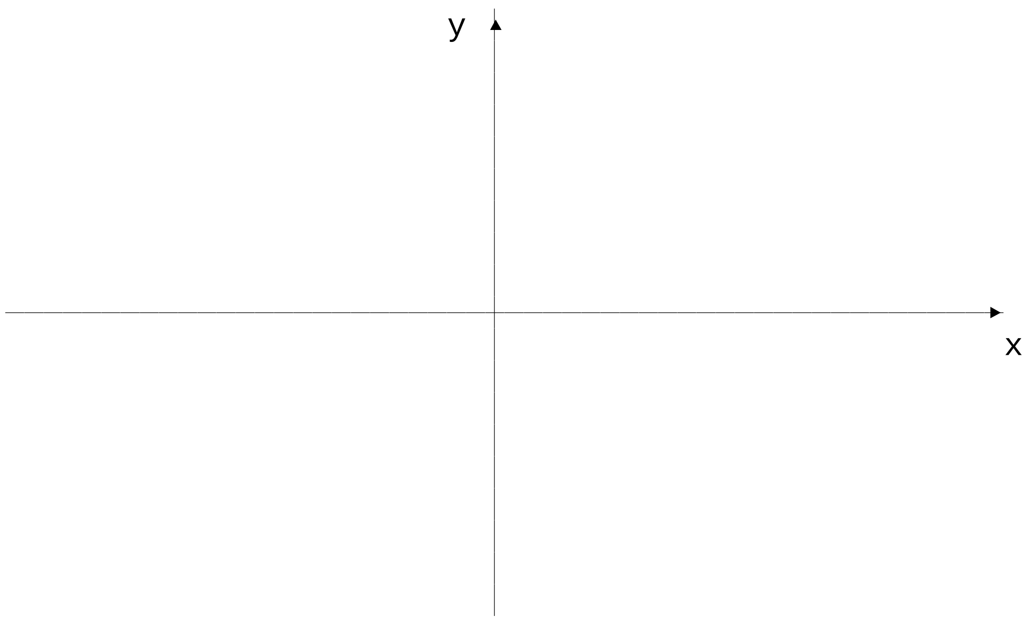
(b) (3 pts.) Determine the open intervals where f is increasing or decreasing.

(c) (3 pts.) Determine the open intervals where f is concave up or concave down.

(d) (3 pts.) List any inflection points or state that there are none.

(e) (3 pts.) Locate any asymptotes.

(f) (5 pts.) Carefully sketch the graph of f below by plotting a few essential points and then connecting the dots appropriately.



Silly 10 point Bonus Problem: Show how to use the Mean Value Theorem to prove the following result: If f is continuous on a closed interval [a,b] with f′(x) > 0 for each x in (a,b), then f is increasing on the closed interval [a,b]. [Say where your work is, for it won’t fit here!]