Student Number: Exam Number:

Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me all the magic on the page. Eschew obfuscation.*

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1. (30 pts.) Basic Derivatives. Provide the first derivative for each of the following. (2 pts./part)

(a) $f(x) = \log_{10}(x)$ $f'(x) =$

(b) $f(x) = 10^x$ $f'(x) =$

(c) $f(x) = x^{10}$ $f'(x) =$

(d) $f(x) = e^x$ $f'(x) =$

(e) $f(x) = \sin(0)$ $f'(x) =$

(f) $f(x) = \sin^{-1}(x)$ $f'(x) =$

(g) $f(x) = \tan(x)$ $f'(x) =$

(h) $f(x) = \sec(x)$ $f'(x) =$

(i) $f(x) = \sec^{-1}(x)$ $f'(x) =$

(j) $f(x) = \cot(x)$ $f'(x) =$

(k) $f(x) = \tan^{-1}(x)$ $f'(x) =$

(l) $f(x) = \ln(x)$ $f'(x) =$

(m) $f(x) = \csc(x)$ $f'(x) =$

(n) $f(x) = \cos^{-1}(x)$ $f'(x) =$

(o) $f(x) = \cos(x)$ $f'(x) =$

2. (30 pts.) Provide each of the following very basic antiderivatives. **Do not forget the arbitrary constant.** 2 points/part.

(a) $\int x^{10}\, dx$ =

(b) $\int 10^{x}\, dx$ =

(c) $\int \frac{1}{x}\, dx$ =

(d) $\int \frac{1}{1+x^2}\, dx$ =

(e) $\int \frac{1}{|x|\sqrt{x^2-1}}\, dx$ =

(f) $\int \frac{1}{\sqrt{1-x^2}}\, dx$ =

(g) $\int \sec^2(x)\, dx$ =

(h) $\int \sec(x)\tan(x)\, dx$ =

(i) $\int \sin(x)\, dx$ =

(j) $\int \cos(x)\, dx$ =

(k) $\int csc(x)\cot(x)\, dx$ =

(l) $\int 10^{10}\, dx$ =

(m) $\int csc^2(x)\, dx$ =

(n) $\int e^{x}\, dx$ =

(o) $\int \ln(2)\, dx$ =

3. (5 pts.) Do you really understand equations with integral signs?? Find the function $g(x)$ that satisfies the following equation

$$\int g(x)\,dx = \sec^2(x) + 2x + C$$

$g(x) =$

4. (5 pts.) Solve the following initial-value problem:

$$\frac{dy}{dx} = \frac{x+1}{\sqrt{x}}, \quad y(1) = 0.$$

5. (20 pts.) Compute the derivatives of the following functions. You may use any of the rules of differentiation that are at your disposal. Do not attempt to simplify the algebra in your answers. (5 pts./part)

(a) $f(x) = \sin(3x^2)e^{\pi x}$

$f'(x) =$

(b) $g(x) = \dfrac{\ln(x)}{x^2+1}$

$g'(x) =$

(c) $h(t) = \sin(\tan^{-1}(3x^2))$

$h'(t) =$

(d) $y = \tan\left(\dfrac{\pi}{3}\right)\theta^5 + \tan\left(\dfrac{\pi}{3}\theta^5\right) + \ln(e^2)$

$\dfrac{dy}{d\theta} =$

6. (15 pts.) Evaluate each of the following integrals or anti-derivatives.

(a) $\int 3x^2 + \frac{1}{x^2+1} + 3\cos(x)\,dx =$

(b) $\int \frac{x^3 + x + 1}{x^2}\,dx =$

(c) $\int \frac{1}{(1 - 4x^2)^{1/2}} + (4x - 2)e^{x^2 - x}\,dx =$

7. (10 pts.) Determine the maximum and minimum values of the function

$$f(x) = 10x^2 - x^4$$

on the interval [-1,5] and where they occur. Before you attempt to find them, explain how you know f(x) actually has both a maximum and a minimum.

8. (10 pts.) Evaluate each of the following limits. L'Hopital may help. Squeezing may help. Nothing may help.

(a) $\lim\limits_{x \to 0} \dfrac{10 - 10\cos(x)}{e^{x} + e^{-x} - 2} =$

(b) $\lim\limits_{t \to \infty} 3t \sin(\dfrac{5}{t}) =$

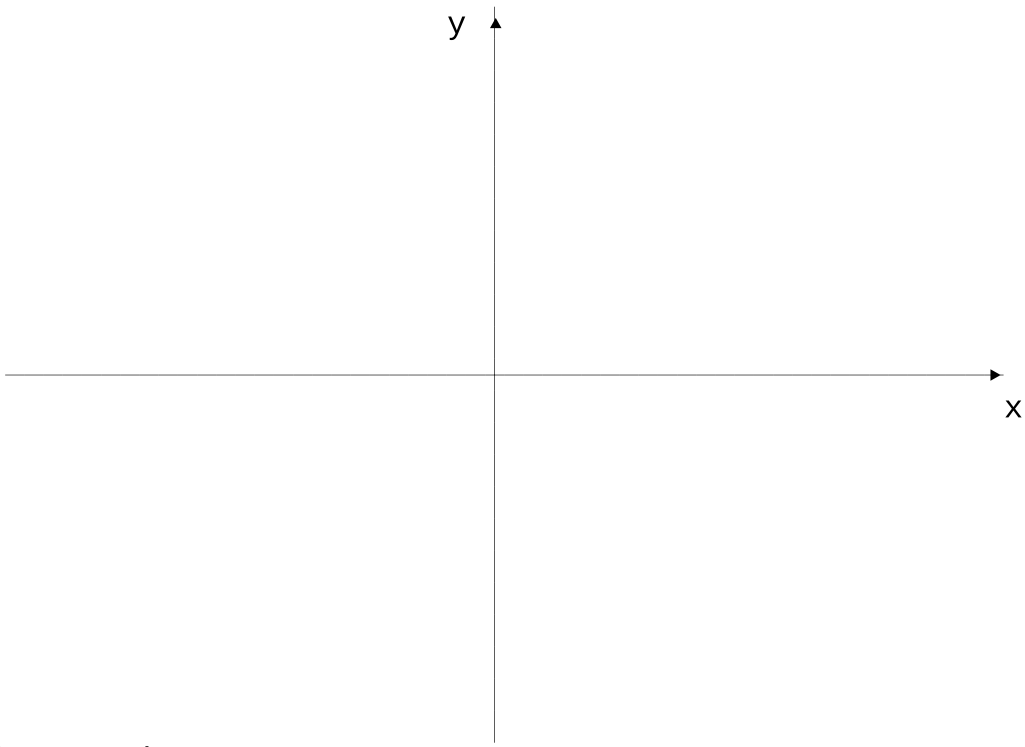
9. (5 pts.) Using only the definition of limit at a point in terms of epsilons and deltas, give a proof that

$$\lim_{x \to 3} \frac{3x^{2} - 27}{x - 3} = 18.$$

10. (10 pts.) A very small rectangular area of 25 square feet is to be fenced. One of the sides will use fencing costing $2.00 per running foot, and the remaining 3 sides will use a hedge costing $1.00 per running foot. Find the dimensions of the rectangle which has the least cost to enclose. Provide a complete enough analysis to convince the doubtful that your extreme value is an absolute minimum.

11. (15 pts.)

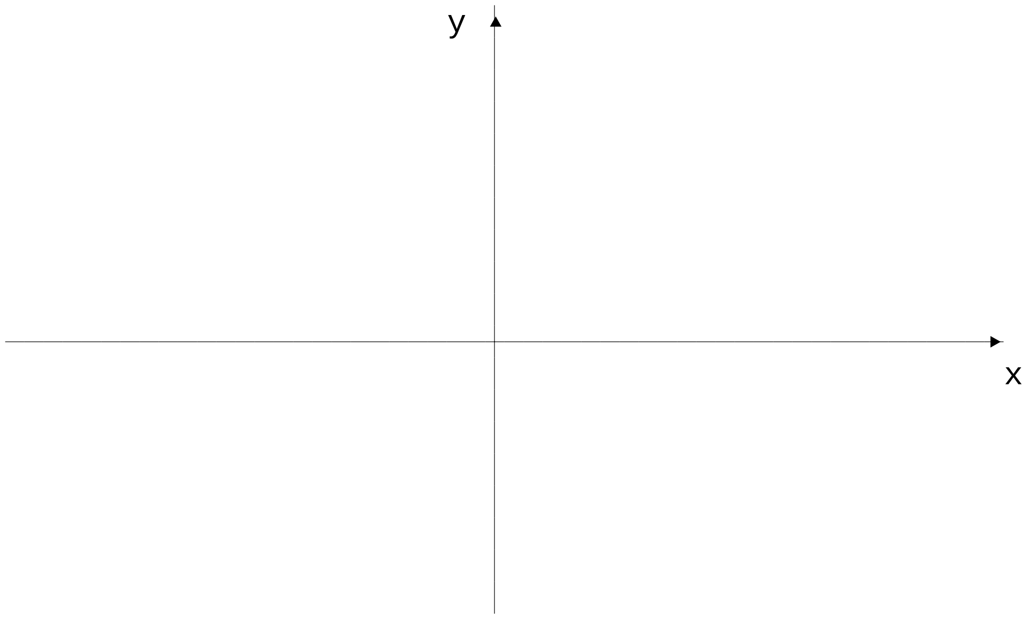
Let $f(x) = 3x^4 - 4x^3$. Analyze f' and f'' and how f behaves as $x \to \pm\infty$. Plot critical points, inflection points, and zeros. Then sketch the graph carefully below:



Analysis (10 pts.):

12. (5 pts.) The radius of a large sphere is measured to be 10 ft, with a possible error of ±0.1 ft. Using differentials, estimate the percent error in the calculated volume of the sphere.

13. (15 pts.) (a) Sketch the curve defined by the parametric equations $x = -1 + \cos(t)$ and $y = 1 - \sin(t)$ with $0 \le t \le 2\pi$ by eliminating the parameter t, and indicate the direction of increasing t.



(b) Write an equation for the line tangent to this curve at the point on the curve where $t_0 = \pi/4$. **Hint:** It helps to compute dy/dx at $t_0 = \pi/4$ without eliminating the parameter.

(c) Compute d^2y/dx^2 at $t_0 = \pi/4$ without eliminating the parameter.

14. (5 pts.) A spherical weather balloon is inflated so that its volume is increasing at a constant rate of 4π cubic feet per minute. How fast is the diameter of the balloon changing when the radius is 9 ft.?
[In case you forgot, $V = (4/3)\pi r^3$.]

15. (8 pts.) (a) State the Mean Value Theorem of Differential Calculus. Use a complete sentence and appropriate notation.

(b) Use the Mean-Value Theorem to show that $1.71 < \sqrt{3} < 1.75$.

16. (12 pts.) (a) Use logarithmic differentiation to differentiate

$$f(x) = x^x \text{ for } x > 0.$$

(b) Compute the following limit:

$\lim_{x \to 0^+} x^x =$

(c) Determine the open intervals where the function $f(x) = x^x$ is increasing or decreasing on its domain, $(0, \infty)$.

Silly 10 point Bonus Problem: Show how to use the Mean Value Theorem to prove the following result: If f is continuous on a closed interval [a,b] with $f'(x) < 0$ for each x in (a,b), then f is decreasing on the closed interval [a,b]. [Say where your work is, for it won't fit here!]