Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.*

1. (10 pts.) Here are five trivial limits to evaluate:

(a) $\lim_{x \to \infty} \frac{1}{\ln x} =$

(b) $\lim_{y \to -\infty} \frac{5y}{|y|} =$

(c) $\lim_{h \to -\infty} \left(5 - \frac{1}{h}\right) =$

(d) $\lim_{x \to \infty} \frac{1}{2 + \sin(x)} =$

(e) $\lim_{h \to \infty} (-2h) =$

2. (15 pts.) Suppose that

$$h(x) = \begin{cases} 2x - x^2 & , \text{if } x > 2 \\ 1 & , \text{if } x = 2 \\ \frac{x^3 - 8}{3x^3} & , \text{if } x < 2 \end{cases}$$

Evaluate each of the following easy limits.

(a) $\lim_{x \to \infty} h(x) =$

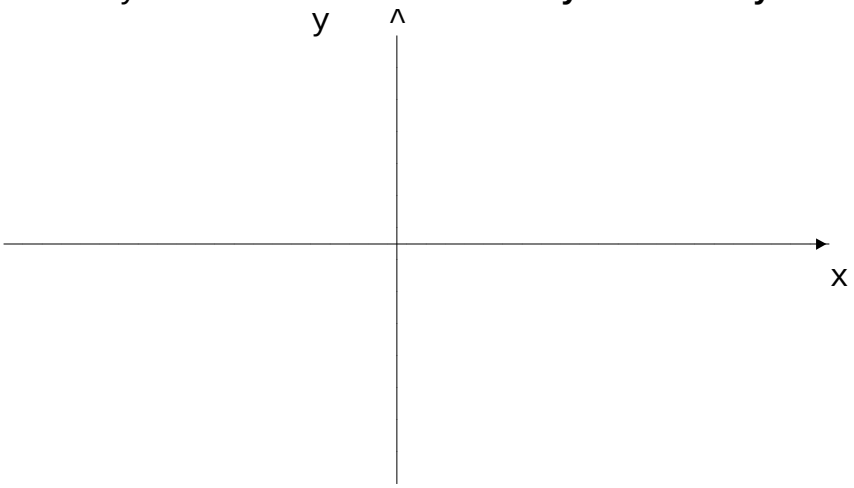
(b) $\lim_{x \to -\infty} h(x) =$

(c) $\lim_{x \to 1} h(x) =$

(d) $\lim_{x \to 2} h(x) =$

(e) $\lim_{x \to 3} h(x) =$

3. (7 pts.) Carefully sketch the function $f(x) = \tan^{-1}(x)$ on the coordinate system below. **Label very carefully**.



Then evaluate the following two limits:

$\lim_{x \to \infty} \tan^{-1}(x) =$ $\lim_{x \to -\infty} \tan^{-1}(x) =$

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4. (8 pts.) If $f(x) = x^2 + 6x$ and $h \neq 0$, then simplifying as much as possible allows us to write

$$\frac{f(x+h) - f(x)}{h} =$$

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5. (10 pts.) (a) Using complete sentences and appropriate notation, provide the precise mathematical definition for

$$\lim_{x \to a} f(x) = L.$$

(b) Using only the mathematical definition of limit, provide a complete proof that

$$\lim_{x \to -3} \frac{x^2 - 9}{x + 3} = -6.$$

6. (25 pts.) For each of the following, find the limit if the limit exists. If the limit fails to exist, say so. Be as precise as possible here. [Work on the back of Page 2 of 4 if you run out of room here.]

(a) $\lim_{x \to 1^-} \frac{x^2-x-2}{x^2-1} =$

(b) $\lim_{x \to -1^+} \frac{x^2-x-2}{x^2-1} =$

(c) $\lim_{x \to 9} \frac{9 - x}{x^{1/2} - 3} =$

(d) $\lim_{x \to 2^-} \frac{4x - 8}{|x - 2|} =$

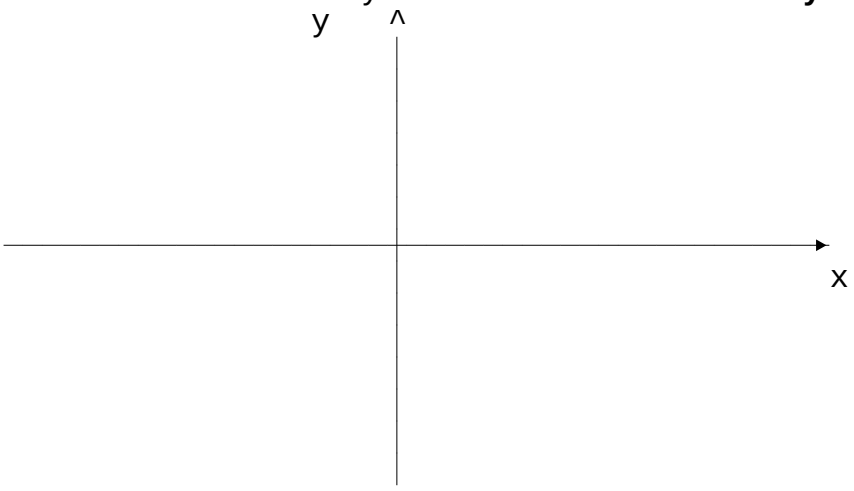
(e) $\lim_{x \to -1} (4 - 36x^2)^{1/5} =$

7. (10 pts.) Evaluate each of the following thorny limits:

(a) $\lim_{x \to +\infty} \frac{\ln(2x)}{\ln(3x)} =$

(b) $\lim_{x \to 0} \frac{(x+4)^{1/2} - 2}{x} =$

8. (15 pts.) (a) (5 pts.) Carefully sketch both f(x) = ln(x) and g(x) = e^x on the coordinate system below. **Label very carefully**.



(b) (10 pts.) Evaluate each of the following limits.

$\lim_{x \to 0^+} \ln(x) =$ $\lim_{x \to \infty} \ln(x) =$

$\lim_{x \to -\infty} e^x =$ $\lim_{x \to \infty} e^x =$

$\lim_{x \to \infty} \left(1 + \frac{1}{2x}\right)^{3x} =$

Silly 10 point Bonus Problem: Provide an ε - δ proof that

$$\lim_{x \to 0} 4x \sin\left(\frac{1}{x}\right) = 0.$$

Say where your work is, for it won't fit here. Hint: The *wiggle* can be controlled.