Read Me First: Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me all the magic on the page. Eschew obfuscation.

1. (25 pts.) Compute the derivatives of the following functions. You may use any of the rules of differentiation that are at your disposal. Do not attempt to simplify the algebra in your answers.

(a) $f(x) = 6x^4 - 12x^{-7} + 8\sec(x)$

$f'(x) =$

(b) $g(x) = (2x^2 - 4x^{-1})\tan(x)$

$g'(x) =$

(c) $h(t) = \dfrac{\sin(t)}{5t^{10}}$

$h'(t) =$

(d) $y = csc(\cot(2\theta + 1))$

$\dfrac{dy}{d\theta} =$

(e) $L(z) = \cos(4z^8) + 4\tan(\dfrac{\pi}{4}) - 4\sin(\dfrac{z}{2})$

$\dfrac{dL}{dz}(z) =$

2. (10 pts.) (a) Using complete sentences and appropriate notation, provide the precise mathematical definition of **continuity** of a function f(x) at a point x = a.

(b) Is there a real number k, that will make the function $f(x)$ defined below continuous at $x = 2$? Either find the value for k and using the definition of continuity, prove that it makes f continuous at $x = 2$, or explain why there cannot be such a number k. Suppose

$$f(x) = \begin{cases} kx^2 , & x \le 2 \\ 2x + k , & x > 2 \end{cases}$$

3. (10 pts.) (a) Using complete sentences and appropriate notation, state the Intermediate Value Theorem.

(b) Prove that if a and b are positive, then the equation

$$\frac{a}{x - 1} + \frac{b}{x - 3} = 0$$

has at least one solution in the interval (1,3).

4. (5 pts.) Evaluate the following limit:

$$\lim_{x \to +\infty} \sin^{-1}\left(\frac{x}{1 - 2x} \right) =$$

5. (5 pts.) Compute $f''(x)$ when

$$f(x) = \sin(3x^2).$$

6. (5 pts.) Find an equation for the tangent line to the graph of

$$y = \tan(4x^2)$$

when $x = \frac{\sqrt{\pi}}{4}$.

7. (10 pts.) (a) Find all values in the interval $[-\pi,\pi]$ at which the graph of f has a horizontal tangent line when $f(x) = x + \cos(x)$.

(b) Using that the following limit represents $f'(a)$ for some function f and some number a, evaluate it:

$$\lim_{x\to 1} \frac{x^7 - 1}{x - 1} =$$

8. (5 pts.) Given that the tangent line to $y = f(x)$ at the point (1,2) passes through the point (-1,-1), find $f'(1)$.

$f'(1) =$

9. (10 pts.) (a) Using complete sentences and appropriate notation, provide the precise mathematical definition for the derivative, $f'(x)$, of a function $f(x)$.

(b) Using only the definition of the derivative as a limit, show all steps of the computation of $f'(x)$ when $f(x) = x^3$.

$f'(x) =$

10. (5 pts.) Pretend f is a magical function that has the property that at x = 3 the tangent line f is actually defined by the equation $y = -5(x - 1) + 2$. Obtain

(a) $f(3) =$ (b) $f'(3) =$

11. (10 pts.) Oil spilled from a ruptured tanker spreads in a circle whose area increases at a constant rate of 6 mi^2/hour. How fast is the radius of the spill increasing when the area is 9 square miles??

Silly 10 point Bonus Problem: Suppose that a function f is differentiable at x_0 and that $f'(x_0) > 0$. Prove that there exists an open interval containing x_0 such that if x_1 and x_2 are any two points in this interval with $x_1 < x_0 < x_2$, then $f(x_1) < f(x_0) < f(x_2)$. // Say where your work is for it won't fit here. Hint: Epsilon antics work wonders. Hint: ε-antics!