Read Me First: Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me all the magic on the page. Eschew obfuscation.

1. (10 pts.) (a) (6 pts.) Find formulas for Δy and the differential dy when $y = x^3$. Label your expressions correctly.

(b) (4 pts.) Use an appropriate local linear approximation formula to estimate $(80.9)^{1/2}$.

2. (10 pts.) (a) Use logarithmic differentiation to find dy/dx when

$$y = (\ln(x))^{\tan(x)}.$$

(b) Find dy/dx by using implicit differentiation when

$$\sin(x^2y^2) = x.$$

3. (10 pts.) Let

$$f(x) = x^5 + x^3 + x.$$

(a) Show that f is one-to-one on $\mathbb{R}$ and confirm the $f(1) = 3$.

(b) Find $(f^{-1})'(3)$.

$(f^{-1})'(3) =$

4. (10 pts.) Write down each of the following derivatives. [2 pts/part.]

(a) $\dfrac{d[\sec^{-1}]}{dx}(x) =$

(b) $\dfrac{d[\cos^{-1}]}{dx}(x) =$

(c) $\dfrac{d[\cot^{-1}]}{dx}(x) =$

(d) $\dfrac{d[csc^{-1}]}{dx}(x) =$

(e) $\dfrac{d[\sin^{-1}]}{dx}(x) =$

5. (10 pts.) Fill in the blanks appropriately.

(a) A function f is increasing on an interval (a, b) if ____________________

whenever $a < x_1 < x_2 < b$.

(b) A function f is decreasing on an interval (a, b) if ____________________

whenever $a < x_1 < x_2 < b$.

(c) A function f is concave down on an interval (a, b) if the derivative of f is ______________________ on (a, b).

(d) A function f is concave up on an interval (a, b) if the derivative of f is ______________________ on (a, b).

(e) A function f has a relative minimum at x_0 if there is an open interval containing x_0 on which ______________________ for each x in both the interval and the domain of f.

6. (5 pts.) Find the following limit by interpreting the expression as an appropriate derivative.

$$\lim_{h \to 0} \frac{\tan^{-1}(1+h) - \frac{\pi}{4}}{h} =$$

7. (5 pts.) The side of the cube is measured to be 10 ft, with a possible error of ±0.1 ft. Using differentials estimate the percent error in the calculated volume.

8. (8 pts.) Assume f is continuous everywhere. If

$$f'(x) = 4x^3 - 36x^2 = 4x^2(x - 9),$$

find all the critical points of f and at each stationary point apply the second derivative test to determine relative extrema, if possible. If the second derivative test fails at a critical point, apply the first derivative test to determine the true state of affairs there.

9. (12 pts.) Evaluate each of the following limits. If a limit fails to exist, say how as specifically as possible.

(a) $\lim_{x \to +\infty} 2x \sin\left(\frac{\pi}{x}\right) =$

(b) $\lim_{x \to 0} \frac{\sin^{-1}(2x)}{x} =$

(c) $\lim_{x \to 0} \frac{x - \tan^{-1}(x)}{x^3} =$

10. (20 pts.) When f is defined by

$$f(x) = \frac{2}{x^2+1} \quad for\ x \in \mathbb{R},$$

$$f'(x) = \frac{-4x}{(x^2+1)^2} \quad \text{and} \quad f''(x) = \frac{16(x-\frac{1}{2})(x+\frac{1}{2})}{(x^2+1)^3}.$$

(a) (3 pts.) What are the critical point(s) of f and what is the value of f at each critical point?

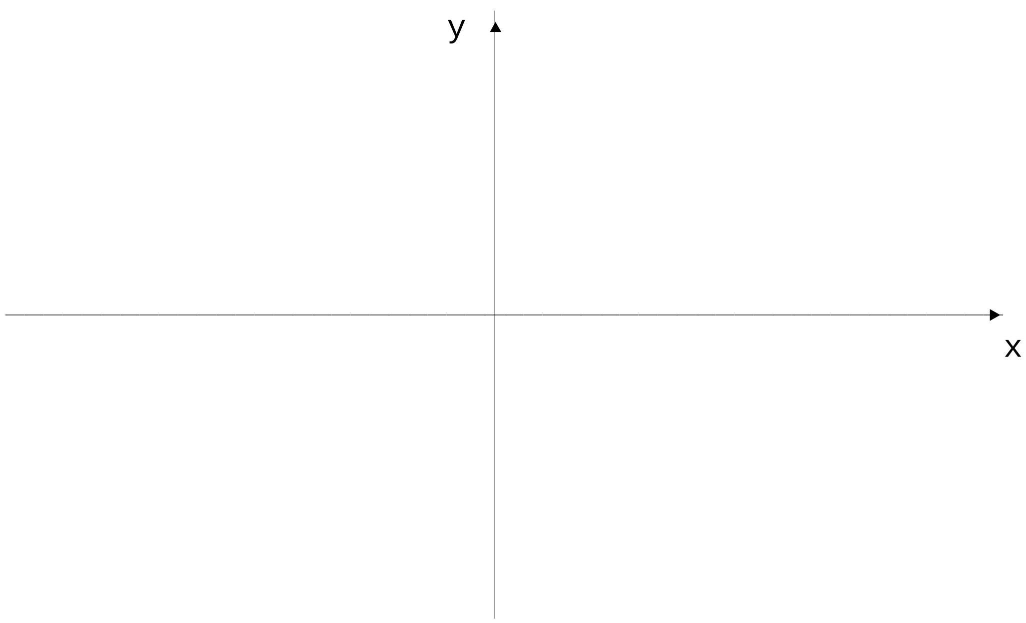
(b) (3 pts.) Determine the open intervals where f is increasing or decreasing.

(c) (3 pts.) Determine the open intervals where f is concave up or concave down.

(d) (3 pts.) List any inflection points or state that there are none.

(e) (3 pts.) Locate any asymptotes.

(f) (5 pts.) Carefully sketch the graph of f below by plotting a few essential points and then connecting the dots appropriately.



Silly 10 point Bonus Problem: Show

$$e^x \geq 1+x \quad if \quad x \geq 0.$$

[Say where your work is, for it won't fit here!]