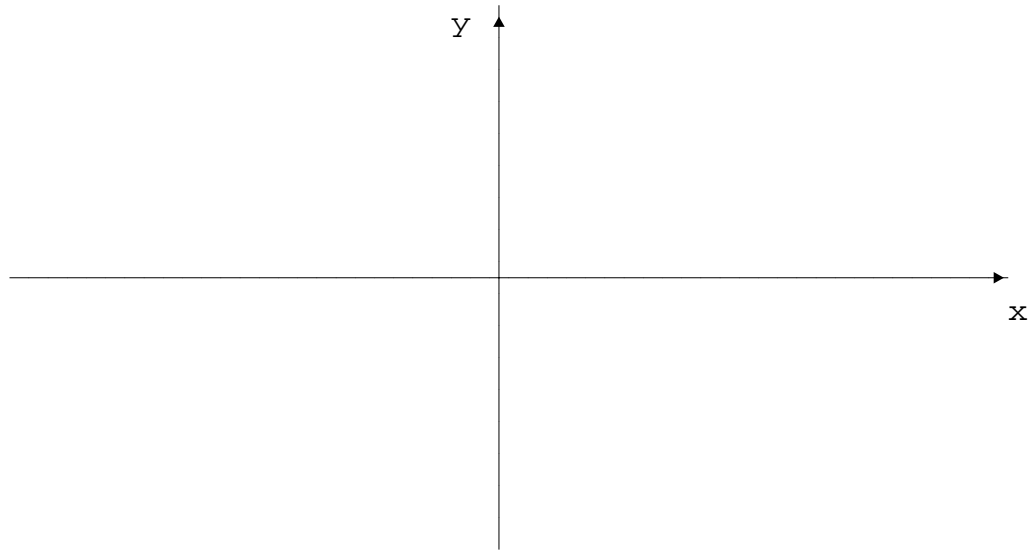

Read Me First: Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , " \Rightarrow " denotes "implies" , and " \Leftrightarrow " denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.

1. (10 pts.) On the coordinate system below sketch the graph of the function g defined below. Label very carefully.

$$g(x) = \begin{cases} x & , \quad x \geq 1 \text{ or } x \leq -1 \\ x^2 & , \quad -1 < x < 1 \end{cases}$$



2. (10 pts.) If $f(x) = 4x^2 - 6x$, find

$$\frac{f(x + h) - f(x)}{h} ,$$

and simplify as much as possible algebraically. Kindly observe that there are no limits of any sort being taken here.

$$\frac{f(x + h) - f(x)}{h} =$$

3. (5 pts.) Using complete sentences and appropriate notation, provide the precise mathematical definition of a **function** f //

4. (10 pts.) Suppose $f(x) = x^2 + 8$ and $g(x) = (x - 8)^{1/2}$. Obtain a formula for each of the following functions and state clearly what their domains are:

(a) $(f + g)(x) =$

$$\text{dom}(f + g) =$$

(b) $(f \cdot g)(x) =$

$$\text{dom}(f \cdot g) =$$

(c) $(f/g)(x) =$

$$\text{dom}(f/g) =$$

(d) $(f - g)(x) =$

$$\text{dom}(f - g) =$$

(e) $(f \circ g) =$

$$\text{dom}(f \circ g) =$$

5. (5 pts.) Express $f(x) = 2 \cdot \cos^2(5x^4)$ as the composition of two functions g and h with $f = g \circ h$, that is find g and h so that $f(x) = (g \circ h)(x)$.

$$g(x) =$$

$$h(x) =$$

Now check your work by correctly computing $(g \circ h)(x)$. [DETAILS REQUIRED!!]

6. (20 pts.) For each of the following, find the limit if the limit exists. If the limit fails to exist, say so. Be as precise as possible here. [Work on the back of page two when you run out of room here.]

$$(a) \quad \lim_{x \rightarrow -5} \frac{x + 5}{x^2 - 25} =$$

$$(b) \quad \lim_{x \rightarrow +5} \frac{x + 5}{x^2 - 25} =$$

$$(c) \quad \lim_{t \rightarrow +\infty} \frac{6 - 14t^3}{2t^3 + 3} =$$

$$(d) \quad \lim_{x \rightarrow +\infty} \frac{7x^4 - 3x^3}{x^3 + 3x^2} =$$

7. (10 pts.) Suppose that

$$h(x) = \begin{cases} x^2 - 3x & , \text{ if } x < -1 \\ 4 & , \text{ if } x = -1 \\ -5x & , \text{ if } x > -1 \end{cases}$$

Evaluate the following limits:

(a) $\lim_{x \rightarrow -1^+} h(x) =$

(b) $\lim_{x \rightarrow -1^-} h(x) =$

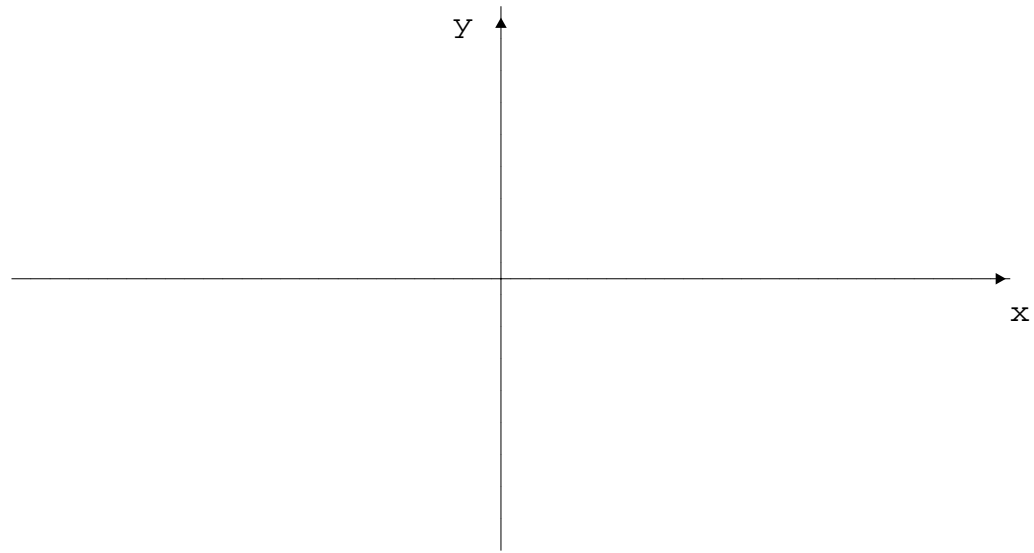
(c) What can you conclude from parts (a) and (b)? Why??

8. (10 pts.) Evaluate each of the following thorny limits:

(a) $\lim_{x \rightarrow \infty} ((x^2 + 8x + 1)^{1/2} - x) =$

(b) $\lim_{x \rightarrow 0} \frac{(x + 4)^{1/2} - 2}{x} =$

9. (10 pts.) Sketch the curve defined by the parametric equations $x = 1 + \cos(t)$ and $y = 1 - \sin(t)$ with $0 \leq t \leq \pi$ by eliminating the parameter, and indicate the direction of increasing t .



10. (5 pts.) Express the following function in piecewise defined form without using absolute values:

$$f(x) = 5|x - 2| - 4x$$

11. (5 pts.) Given $\lim_{x \rightarrow a} [2f(x) - 2g(x)] = 3$ and $\lim_{x \rightarrow a} g(x) = -8$,

$$\lim_{x \rightarrow a} f(x) =$$

$$\lim_{x \rightarrow a} [g(x)]^{1/3} =$$