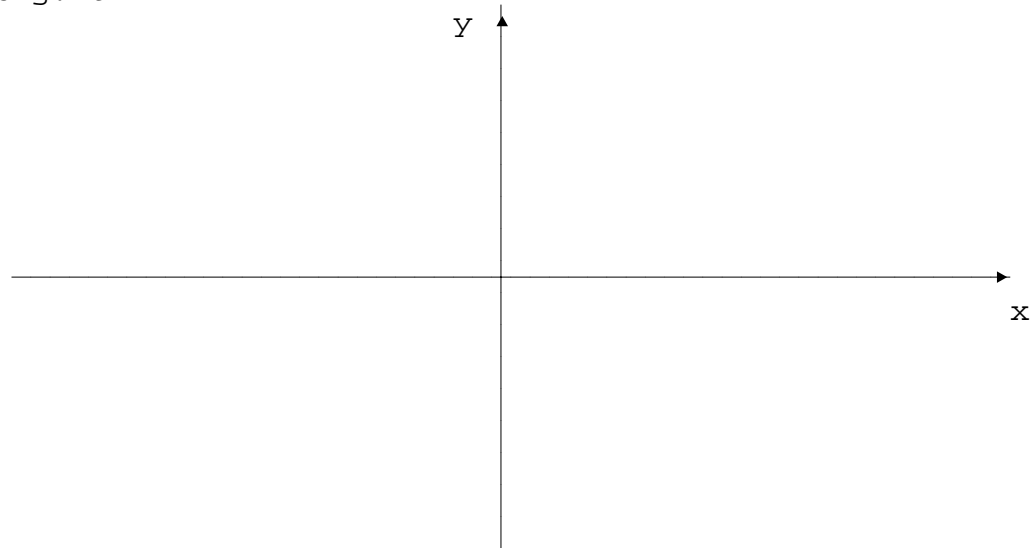

Read Me First: Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , " \Rightarrow " denotes "implies" , and " \Leftrightarrow " denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.

1. (10 pts.) On the coordinate system below sketch the graph of the function g defined below. Label very carefully.

$$g(x) = \begin{cases} x - 1 & , \quad x \geq 1 \text{ or } x \leq -1 \\ 1 - x^2 & , \quad -1 < x < 1 \end{cases}$$

See c1-t1b-g.htm



2. (10 pts.) If $f(x) = 6x^2 - 4x$, find

$$\frac{f(x + h) - f(x)}{h} ,$$

and simplify as much as possible algebraically. Kindly observe that there are no limits of any sort being taken here.

$$\begin{aligned} \frac{f(x + h) - f(x)}{h} &= \frac{[6(x+h)^2 - 4(x+h)] - [6x^2 - 4x]}{h} \\ &= 12x - 4 + 6h \quad \text{for } h \neq 0 \end{aligned}$$

after the algebraic dust settles.

3. (5 pts.) Using complete sentences and appropriate notation, provide the precise mathematical definition of a **function** f //

A function f is a rule that associates a unique output with each input. If the input is denoted by x , then the output is denoted by $f(x)$. [Note: The uniqueness of the output is critical.] *Definition 1.1.2.*

4. (10 pts.) Suppose $f(x) = x^2 + 3$ and $g(x) = (x - 3)^{1/2}$. Obtain a formula for each of the following functions and state clearly what their domains are:

$$(a) \quad (f + g)(x) = x^2 + 3 + (x - 3)^{1/2}$$

$$\text{dom}(f + g) = [3, \infty)$$

$$(b) \quad (f \cdot g)(x) = (x^2 + 3) \cdot (x - 3)^{1/2}$$

$$\text{dom}(f \cdot g) = [3, \infty)$$

$$(c) \quad (f/g)(x) = (x^2 + 3)/(x - 3)^{1/2}$$

$$\text{dom}(f/g) = (3, \infty)$$

$$(d) \quad (f - g)(x) = x^2 + 3 - (x - 3)^{1/2}$$

$$\text{dom}(f - g) = [3, \infty)$$

$$(e) \quad (f \circ g) = f(g(x)) = ((x - 3)^{1/2})^2 + 3 = x$$

$$\text{dom}(f \circ g) = \{ x \in \text{dom}(g) \mid g(x) \in \text{dom}(f) \}$$

$$= \{ x \mid x \in \text{dom}(g) \text{ and } g(x) \in \text{dom}(f) \}$$

$$= \{ x \mid x - 3 \geq 0 \}$$

$$= [3, +\infty)$$

5. (5 pts.) Express $f(x) = 5 \cdot \cos^2(2x^4)$ as the composition of two functions g and h with $f = g \circ h$, that is find g and h so that $f(x) = (g \circ h)(x)$.

$$g(x) = 5 \cdot \cos^2(x)$$

$$h(x) = 2x^4$$

Now check your work by correctly computing $(g \circ h)(x)$. [DETAILS REQUIRED!!] $(g \circ h)(x) = g(h(x)) = 5 \cdot \cos^2(h(x)) = 5 \cdot \cos^2(2x^4)$

6. (20 pts.) For each of the following, find the limit if the limit exists. If the limit fails to exist, say so. Be as precise as possible here. [Work on the back of page two when you run out of room here.]

$$(a) \quad \lim_{x \rightarrow -6} \frac{x + 6}{x^2 - 36} = \lim_{x \rightarrow -6} \frac{1}{x - 6} = -1/12$$

$$(b) \quad \lim_{x \rightarrow +6} \frac{x + 6}{x^2 - 36} = \lim_{x \rightarrow +6} \frac{1}{x - 6} \quad \text{fails to exist.}$$

$$(c) \quad \lim_{t \rightarrow +\infty} \frac{14 - 6t^7}{2t^3 + 3} = \lim_{t \rightarrow +\infty} \frac{t^7[14t^{-7} - 6]}{t^3[2 + 3t^{-3}]} = -\infty$$

$$(d) \quad \lim_{x \rightarrow +\infty} \frac{7x^3 - 3x^5}{x^5 + 3x^2} = \lim_{x \rightarrow +\infty} \frac{x^5[7x^{-2} - 3]}{x^5[1 + 3x^{-3}]} = -3$$

7. (10 pts.) Suppose that

$$h(x) = \begin{cases} -2x^2 - 3x & , \text{ if } x < -1 \\ 11 & , \text{ if } x = -1 \\ -5x & , \text{ if } x > -1 \end{cases}$$

Evaluate the following limits:

$$(a) \quad \lim_{x \rightarrow -1^+} h(x) = \lim_{x \rightarrow -1^+} (-5x) = 5$$

$$(b) \quad \lim_{x \rightarrow -1^-} h(x) = \lim_{x \rightarrow -1^-} (-2x^2 - 3x) = 1$$

(c) What can you conclude from parts (a) and (b)? Why??

Since the left and right limits are not the same, $\lim_{x \rightarrow -1} h(x)$ fails to exist.
 [You might observe that the function h fails to be continuous at $x = -1$. The important thing at this stage is that you recognize that the limit does not exist since the two one-sided limits exist and have different values.]

8. (10 pts.) Evaluate each of the following thorny limits:

$$(a) \quad \lim_{x \rightarrow \infty} ((x^2 + 18x + 1)^{1/2} - x) = \lim_{x \rightarrow \infty} \frac{18x + 1}{(x^2 + 18x + 1)^{1/2} + x}$$

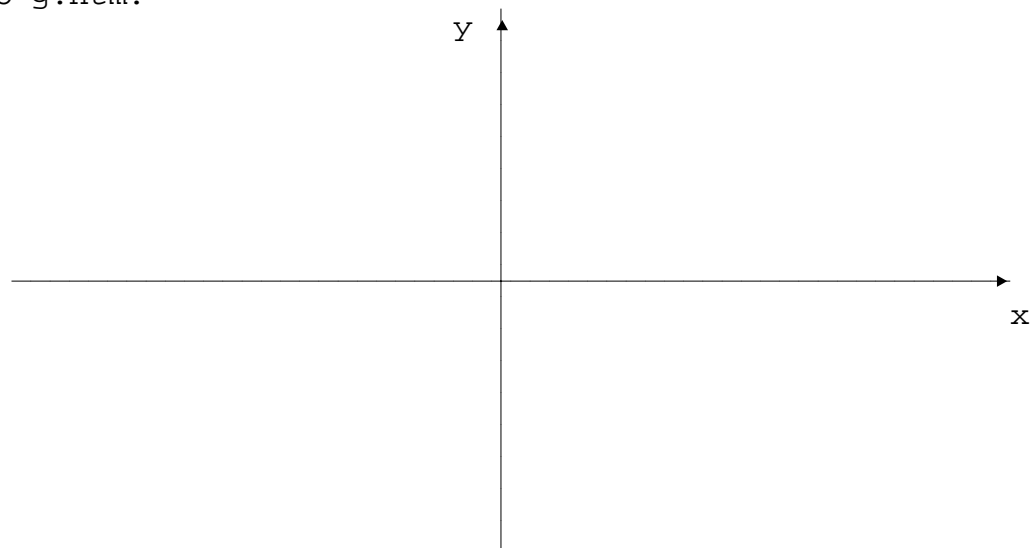
$$= \lim_{x \rightarrow \infty} \frac{18x + 1}{|x|(1 + 18x^{-1} + x^{-2})^{1/2} + x} = 18/2 = 9$$

$$(b) \quad \lim_{x \rightarrow 0} \frac{(x + 9)^{1/2} - 3}{x} = \lim_{x \rightarrow 0} \frac{1}{(x + 9)^{1/2} + 3} = 1/6$$

The key piece of algebraic magic above is captured in the prestidigitatation of rationalizing the numerator. Why would one dream of doing that?? Look at the form ...

9. (10 pts.) Sketch the curve defined by the parametric equations $x = -1 + \cos(t)$ and $y = 1 - \sin(t)$ with $\pi \leq t \leq 2\pi$ by eliminating the parameter, and indicate the direction of increasing t . //FIXMr. Pythagoras allows us to eliminate the parameter t and write $(x - (-1))^2 + (y - 1)^2 = 1$. This wheelie is done in the negative sense and only involves the top half of the circle. [Pay attention to the parameter set!!]

See c1-t1b-g.htm.



10. (5 pts.) Express the following function in piecewise defined form without using absolute values:

$$f(x) = 4|x - 2| - 5x$$

$$\begin{aligned} f(x) &= \begin{cases} 4(x - 2) - 5x & , \text{ if } x \geq 2 \\ 4(2 - x) - 5x & , \text{ if } x < 2 \end{cases} \\ &= \begin{cases} -x - 8 & , \text{ if } x \geq 2 \\ -9x + 8 & , \text{ if } x < 2 \end{cases} \end{aligned}$$

11. (5 pts.) Given $\lim_{x \rightarrow a} [2f(x) - 2g(x)] = -8$ and $\lim_{x \rightarrow a} g(x) = 4$,

$$\lim_{x \rightarrow a} f(x) = (1/2) \lim_{x \rightarrow a} \{ [2f(x) - 2g(x)] + 2g(x) \}$$

$$= (1/2)((-8) + (2) \cdot (4)) = 0/2 = 0$$

$$\lim_{x \rightarrow a} [g(x)]^{1/3} = [4]^{1/3}$$