
Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇨" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.*

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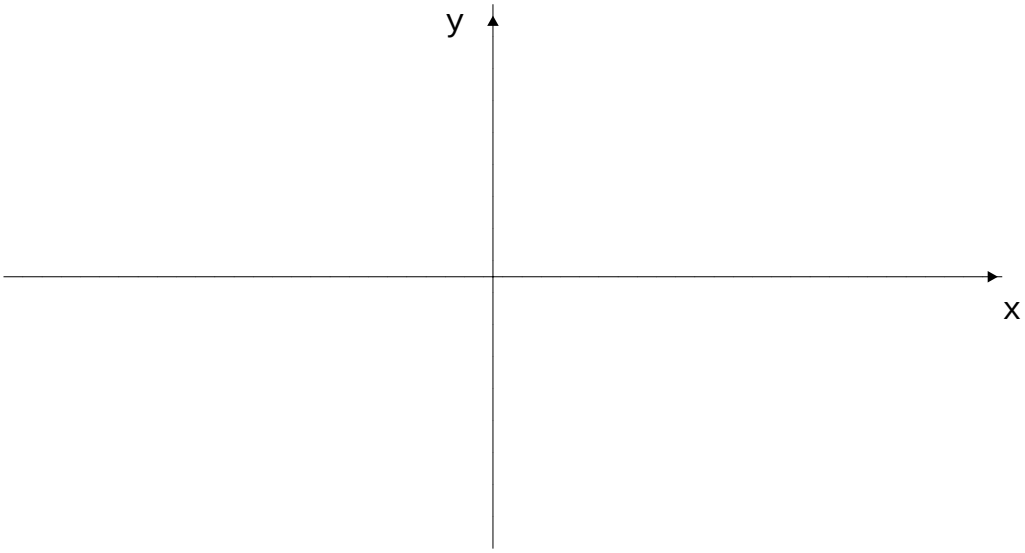
1. (10 pts.) On the coordinate system below sketch the graph of the function g defined below. Label very carefully.

$$g(x) = \begin{cases} x - 1 & , \quad x \geq 1 \text{ or } x \leq -1 \\ 1 - x^2 & , \quad -1 < x < 1 \end{cases}$$



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2. (10 pts.) If $f(x) = 6x^2 - 4x$, find

$$\frac{f(x + h) - f(x)}{h},$$

and simplify as much as possible algebraically. Kindly observe that there are no limits of any sort being taken here.

$$\frac{f(x + h) - f(x)}{h} =$$

3. (5 pts.) Using complete sentences and appropriate notation, provide the precise mathematical definition of a **function** f //

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4. (10 pts.) Suppose $f(x) = x^2 + 3$ and $g(x) = (x - 3)^{1/2}$. Obtain a formula for each of the following functions and state clearly what their domains are:

(a) $(f + g)(x) =$

dom(f + g) =

(b) $(f \cdot g)(x) =$

dom(f · g) =

(c) $(f/g)(x) =$

dom(f/g) =

(d) $(f - g)(x) =$

dom(f - g) =

(e) $(f \circ g) =$

dom(f ∘ g) =

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5. (5 pts.) Express $f(x) = 5 \cdot \cos^2(2x^4)$ as the composition of two functions g and h with f = g∘h, that is find g and h so that f(x) = (g∘h)(x).

g(x) =

h(x) =

Now check your work by correctly computing (g∘h)(x). [DETAILS REQUIRED!!]

6. (20 pts.) For each of the following, find the limit if the limit exists. If the limit fails to exist, say so. Be as precise as possible here. [Work on the back of page two when you run out of room here.]

(a) $\lim_{x \to -6} \dfrac{x + 6}{x^2 - 36} =$

(b) $\lim_{x \to +6} \dfrac{x + 6}{x^2 - 36} =$

(c) $\lim_{t \to +\infty} \dfrac{14 - 6t^7}{2t^3 + 3} =$

(d) $\lim_{x \to +\infty} \dfrac{7x^3 - 3x^5}{x^5 + 3x^2} =$

7. (10 pts.) Suppose that

$$h(x) = \begin{cases} -2x^2 - 3x & \text{, if } x < -1 \\ 11 & \text{, if } x = -1 \\ -5x & \text{, if } x > -1 \end{cases}$$

Evaluate the following limits:

(a) $\lim_{x \to -1^+} h(x) =$

(b) $\lim_{x \to -1^-} h(x) =$

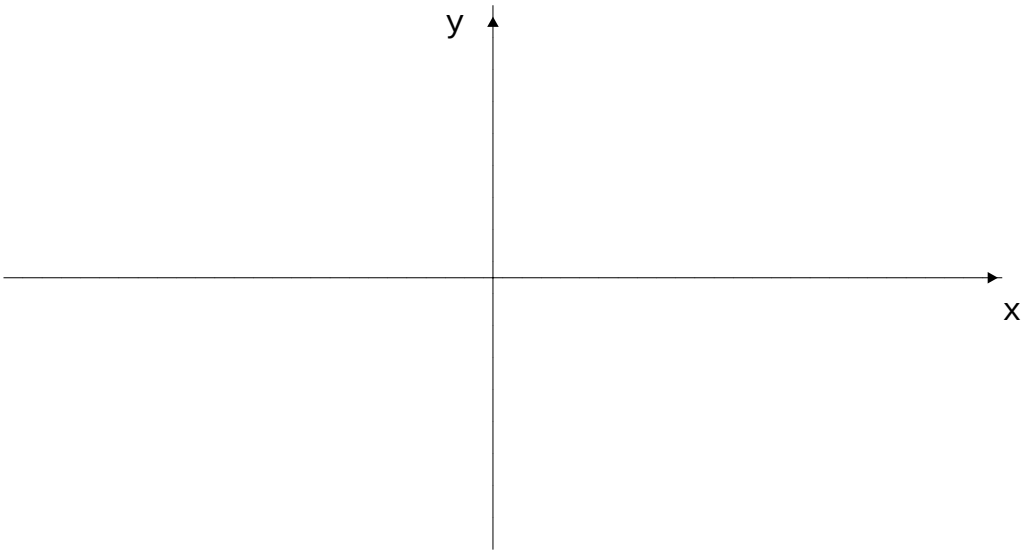
(c) What can you conclude from parts (a) and (b)? Why??

8. (10 pts.) Evaluate each of the following thorny limits:

(a) $\lim_{x \to \infty} ((x^2 + 18x + 1)^{1/2} - x) =$

(b) $\lim_{x \to 0} \dfrac{(x + 9)^{1/2} - 3}{x} =$

9. (10 pts.) Sketch the curve defined by the parametric equations $x = -1 + \cos(t)$ and $y = 1 - \sin(t)$ with $\pi \leq t \leq 2\pi$ by eliminating the parameter, and indicate the direction of increasing t.



10. (5 pts.) Express the following function in piecewise defined form without using absolute values:

$$f(x) = 4|x - 2| - 5x$$

11. (5 pts.) Given $\lim_{x \to a} [2f(x) - 2g(x)] = -8$ and $\lim_{x \to a} g(x) = 4$,

$\lim_{x \to a} f(x) =$

$\lim_{x \to a} [g(x)]^{1/3} =$