Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me all the magic on the page.*

1. (20 pts.) Compute the derivatives of the following functions. You may use any of the rules of differentiation that are at your disposal. Do not attempt to simplify the algebra in your answers.

(a) $f(x) = 4x^{-5} - 3x^{6} + 7\cdot\cot(x)$

$f'(x) =$

(b) $g(x) = (x^{3} - 4x^{-1})\sin(x)$

$g'(x) =$

(c) $h(t) = \dfrac{3\cot(t)}{2t^{5} - \sec(t)}$

$h'(t) =$

(d) $L(x) = 4\cdot\tan^{2}(x^{3})$

$L'(x) =$

(e) $y = \sin(21x) + 5\cos(7x) + \sec(\pi/4)$

$\dfrac{dy}{dx} =$

2. (10 pts.) (a) Using complete sentences and appropriate notation, state the Intermediate Value Theorem.

(b) Can the Intermediate Value Theorem be used to show the equation

$$(2x - 1)/(x^2 - 4x - 5) = 0$$

has a solution in the interval [0,4]? Explain completely.

[Hint: Let $f(x) = (2x - 1)/(x^2 - 4x - 5)$ on [0,4]. Discuss appropriate properties of f and then draw a suitable conclusion. Kindly note that you are not being asked whether there is a solution. What is in question is whether the magic theorem implies there is a solution.]

3. (10 pts.) (a) Using complete sentences and appropriate notation, provide the precise mathematical definition for the derivative, $f'(x)$, of a function $f(x)$.

(b) Using the definition of the derivative as a limit, show all steps of the computation of $f'(x)$ when $f(x) = 2x^2 - 4x$.

$f'(x) =$

4. (10 pts.) Using complete sentences and appropriate notation, provide the precise mathematical definitions for each of the following items:

(a) $\lim_{x \to a} f(x) = L$ [Hint: This involves ε and δ.]//

(b) **Continuity** of a function f(x) at a point x = a //

5. (5 pts.) Find a value for the constant k, if possible, that will make the function h(x) defined below continuous at x = 5. If you find such a k, using the definition, verify the continuity of h(x) at x = 5. Suppose that

$$h(x) = \begin{cases} 2(x - 5)/(x^2 - 4x - 5), & x \neq 5 \\ 10 \cdot k, & x = 5. \end{cases}$$

6. (5 pts.) Evaluate the following limit. To obtain full credit, you must show all essential steps correctly in a chain of equations.

$$\lim_{\theta \to 0} \frac{\tan(8\pi^3\theta)}{\sin(3\pi\theta)} =$$

7. (5 pts.) Compute $f''(x)$ when $f(x) = \sin(4x^3)$. Label your expressions correctly or else.

8. (5 pts.) Obtain an equation for the line tangent to the graph of $f(x) = \cos(2x)$ at $x_0 = \pi/6$.

9. (5 pts.) Show an evaluation of the following limit that is completely correct. You will need to build a suitable inequality to provide a complete solution.

$$\lim_{x \to 0} x^2\sin(4/x^5) =$$

10. (5 pts.) Find all points in the interval $[-2\pi,2\pi]$ where the graph of $f(x) = x - 2\cos(x)$ has a horizontal tangent line.

11. (5 pts.) Obtain the following limit. This is easy if you grok the definition of the derivative.

$$\lim_{h \to 0} \frac{\sec((\pi/3) + h) - \sec(\pi/3)}{h} =$$

12. (5 pts.) Pretend f is a magical function that has the property that at $x = \pi$ the tangent line to the graph of f is actually defined by the equation $y = -\pi x + 3\pi^2$. Obtain the following:

(a) $f'(\pi) =$

(b) $f(\pi) =$

13. (5 pts.) Is the function g defined below differentiable at x = 0 ?? Prove your guess is correct.

$$g(x) = \begin{cases} (x - 1)^2 & , \quad x \geq 0 \\ -2x + 1 & , \quad x < 0 \end{cases}$$

14. (5 pts.) Give an ε - δ proof that $\lim_{x \to 2}(7x - 2) = 12$.