Student Number: Exam Number:

Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Show me all the magic on the page. Eschew obfuscation.*

1. (36 pts.) Provide each of the following very basic antiderivatives. **Do not forget the arbitrary constant of integration.** 3 points/part.

(a) $\int \sec^2(x)\, dx$ =

(b) $\int \frac{1}{1+x^2}\, dx$ =

(c) $\int \cos(x)\, dx$ =

(d) $\int \frac{1}{|x|\sqrt{x^2-1}}\, dx$ =

(e) $\int e^x\, dx$ =

(f) $\int \frac{1}{\sqrt{1-x^2}}\, dx$ =

(g) $\int x^{10}\, dx$ =

(h) $\int \frac{1}{x}\, dx$ =

(i) $\int \frac{1}{x^{10}}\, dx$ =

(j) $\int 2^x\, dx$ =

(k) $\int \sin(x)\, dx$ =

(l) $\int csc(x)\cot(x)\, dx$ =

2. (10 pts.) Obtain an equation for the tangent line to the curve defined by

$$x + y + xy^2 = 3$$

at the point (2,-1) which is actually on the curve.

3. (10 pts.) Suppose that a one-to-one function f has a tangent line given by

$$y = -5x + 15$$

at the point (1,10).

(a) Then

$(f^{-1})'(10) =$

(b) Using a local linear approximation, estimate the value of $f(.9)$.

$f(.9) \approx$

4. (12 pts.) Determine the maximum and minimum values of the function

$$f(x) = 8x^2 - x^4$$

on the interval [1,3] and where they occur. Before you attempt to find them, explain how you know $f(x)$ actually has both a maximum and a minimum.

5. (10 pts.) Solve the following initial-value problem:

$$\frac{dy}{dt} = \sin(t) + 1, \quad y\left(\frac{\pi}{3}\right) = \frac{1}{2}.$$

6. (12 pts.) (a) State the Mean-Value Theorem of Differential Calculus. Use a complete sentence and appropriate notation.

(b) Let $f(x) = x^2 - x$.

Then f satisfies the hypotheses of the Mean-Value Theorem on the interval [-3,5]. Find all value(s) c in the interval [-3,5] where the conclusion to the Mean-Value Theorem is satisfied.

7. (10 pts.) A box with a square base, vertical sides, and an open top is made from 300 square feet of material. Find the dimensions yielding the greatest volume. Merely giving the numbers does not suffice. Give an explicit definition of the function you are optimizing and its domain. Prove your maximum is really the maximum.

8. (18 pts.) Evaluate each of the following integrals or anti-derivatives. You may need to do additional algebra or a substitution.

(a) $\int \frac{3}{x^4} + \frac{4}{x^2+1} + \frac{5\cos(x)}{\sin(x)}\, dx =$

(b) $\int \frac{x^3 - 2x^2 + 1}{x^3}\, dx =$

(c) $\int \frac{x}{\sqrt{1 - 9x^2}} - \cos(x)\, e^{\sin(x)}\, dx =$

9. (18 pts.) Compute the derivatives of the following functions. You may use any of the rules of differentiation that are at your disposal. Do not attempt to simplify the algebra in your answers.

(a) $f(x) = \tan(3x^2)\ln(\pi x)$

$f'(x) =$

(b) $g(x) = \frac{e^{x^2}}{\sec^{-1}(x)}$

$g'(x) =$

(c) $h(t) = \sin(\tan^{-1}(x^2))$

$h'(t) =$

10. (12 pts.) Let $f(x) = x^3e^{-x}$

(a) Find each of the following limits:

$$\lim_{x \to \infty} (x^3e^{-x}) =$$

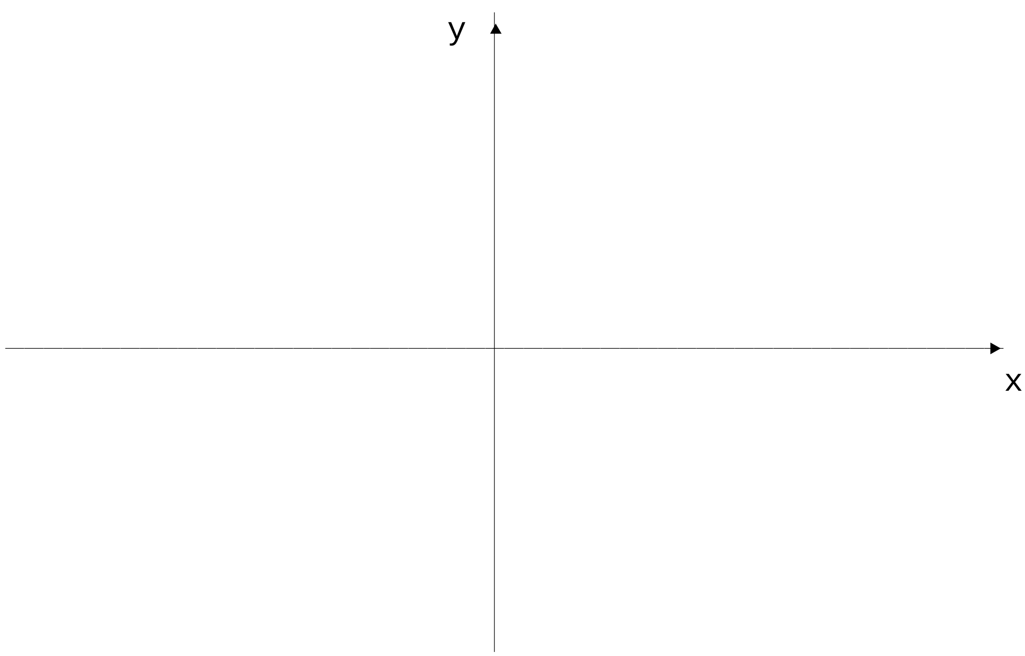
$$\lim_{x \to -\infty} (x^3e^{-x}) =$$

(b) Now find the absolute maximum and minimum values of f, if any, on the interval $(-\infty, \infty)$, and state where those values occur.

11. (12 pts.) Very carefully sketch the function

$$f(x) = 4x^3 - 3x^4 = -3x^3(x - \frac{4}{3})$$

Do this only after you have completely analyzed its behavior.



Analysis:

12. (10 pts.) Evaluate each of the following limits. L'Hopital may help. Squeezing may help. Nothing may help.

(a) $\lim_{x \to 0} \dfrac{x - \tan^{-1}(x)}{x^3} =$

(b) $\lim_{x \to \pi/2^-} \sec(3x)\cos(5x) =$

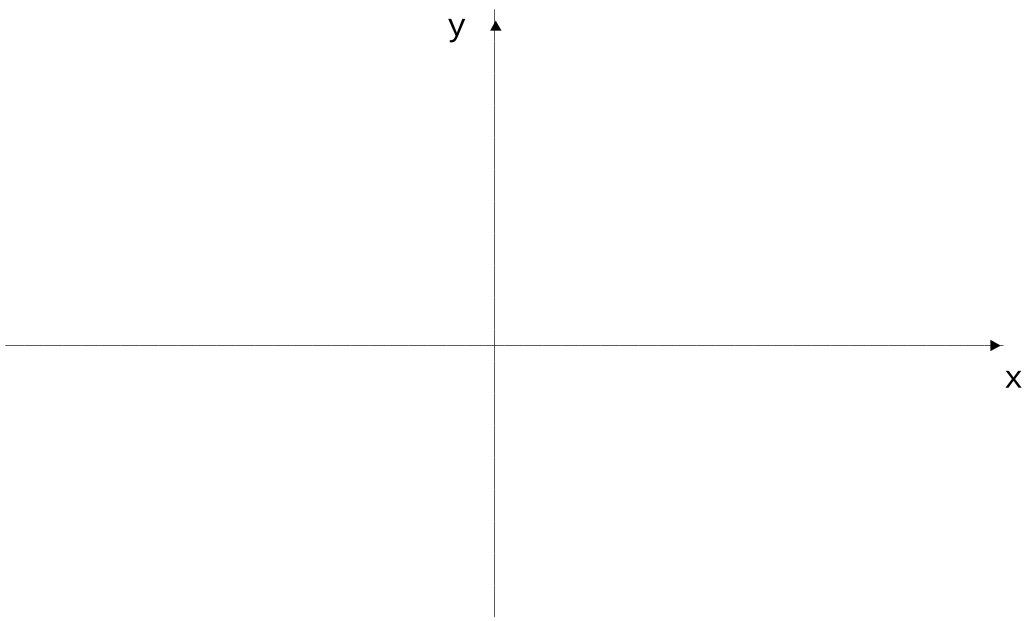
13. (10 pts.) (a) Suppose that we have

$$x = \sec(t) \quad \textit{and} \quad y = \tan(t).$$

Find dy/dx and d^2y/dx^2 at $t = \pi/3$.

(b) Sketch the curve by eliminating the parameter, and indicate the direction of increasing t when the curve is defined parametrically by

$$x = \sec^2(t)\ ,\ y = \tan^2(t) \ \textit{for}\ -\pi/2 < t \le 0.$$



14. (10 pts.) (a) Use logarithmic differentiation to differentiate

$$f(x) = x^x = e^{x\ln(x)} \quad \textit{for } x > 0.$$

(b) Determine the open intervals where the function $f(x) = x^x$ is increasing or decreasing on its domain, $(0\ ,\ \infty)$.

15. (10 pts.) (a) Using complete sentences and appropriate notation, provide the precise mathematical definition for the derivative, $f'(x)$, of a function $f(x)$.

(b) Using only the definition of the derivative as a limit, show all steps of the computation of $f'(x)$ when $f(x) = x^{1/2}$. [Using 'Power Rule' will result in no points being awarded.]

$f'(x) =$

10 Bonkers Bonus Points: Show how to use the Mean-Value Theorem to prove the following result: If f is continuous on a closed interval [a,b] with f′(x) > 0 for each x in (a,b), then f is increasing on the closed interval [a,b]. [Say where your work is, for it won't fit here!]