
Read Me First: Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals", " \Rightarrow " denotes "implies", and " \Leftrightarrow " denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.

1. (10 pts.) Here are five trivial limits to evaluate:

(a) $\lim_{x \rightarrow 0^-} \frac{x}{|x|} =$

(b) $\lim_{z \rightarrow 1^-} \frac{1}{1-z} =$

(c) $\lim_{x \rightarrow -1} (x^3 + x^2 + x)^{101} =$

(d) $\lim_{x \rightarrow \infty} \frac{2}{\ln(x)} =$

(e) $\lim_{h \rightarrow \infty} \left(6 - \frac{1}{h} \right) =$

2. (15 pts.) Suppose that

$$h(x) = \begin{cases} 2x - x^3, & \text{if } x < 2 \\ 1, & \text{if } x = 2 \\ \frac{x^2 - 8}{3x^3}, & \text{if } x > 2 \end{cases}$$

Evaluate each of the following easy limits.

(a) $\lim_{x \rightarrow \infty} h(x) =$

(b) $\lim_{x \rightarrow -\infty} h(x) =$

(c) $\lim_{x \rightarrow 1} h(x) =$

(d) $\lim_{x \rightarrow 2} h(x) =$

(e) $\lim_{x \rightarrow 3} h(x) =$

3. (25 pts.) For each of the following, find the limit if the limit exists. If the limit fails to exist, say so. Be as precise as possible here. [Work on the back of Page 2 of 4 if you run out of room here.]

$$(a) \quad \lim_{x \rightarrow 3} \frac{x^2 - 2x}{x + 1} =$$

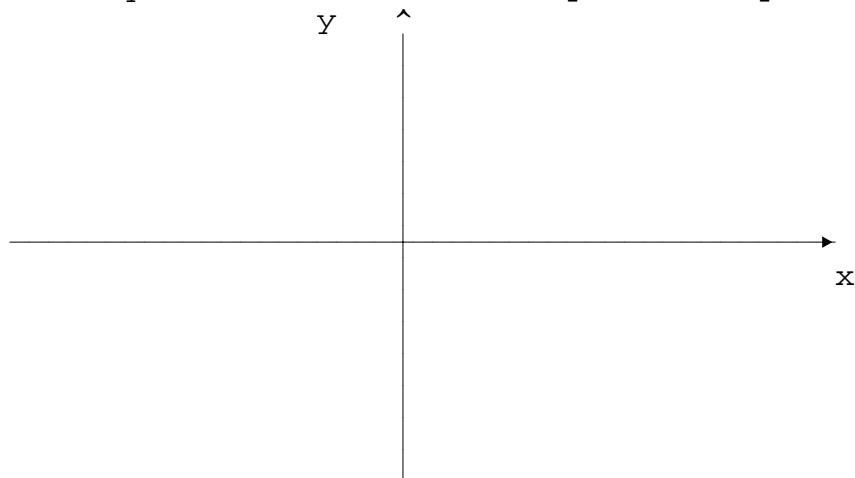
$$(b) \quad \lim_{x \rightarrow 1^+} \frac{x^4 - 1}{x - 1} =$$

$$(c) \quad \lim_{y \rightarrow 1} \frac{1 - y}{1 - \sqrt{y}} =$$

$$(d) \quad \lim_{x \rightarrow 1^-} \frac{4x - 4}{|x - 1|} =$$

$$(e) \quad \lim_{x \rightarrow -\infty} \frac{\sqrt{5x^2 - 2}}{x + 3} =$$

4. (7 pts.) Carefully sketch the function $f(x) = \tan^{-1}(x)$ on the coordinate system below. **Label very carefully.**



Then evaluate the following two limits:

$$\lim_{x \rightarrow \infty} \tan^{-1}(x) =$$

$$\lim_{x \rightarrow -\infty} \tan^{-1}(x) =$$

5. (8 pts.) If $f(x) = x^3$ and $h \neq 0$, then simplifying as much as possible allows us to write

$$\frac{f(x+h) - f(x)}{h} =$$

6. (10 pts.) (a) Using complete sentences and appropriate notation, provide the precise mathematical definition for

$$\lim_{x \rightarrow a} f(x) = L.$$

(b) Using only the mathematical definition of limit, provide a complete proof that

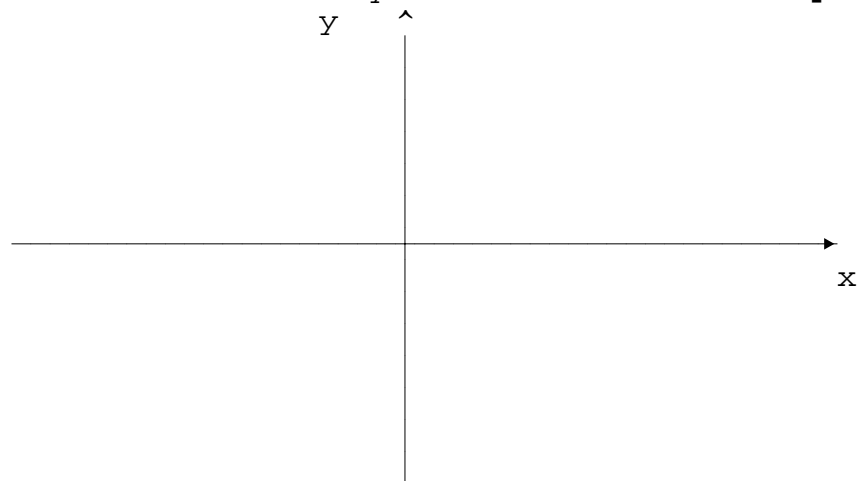
$$\lim_{x \rightarrow -1} \frac{4x^2 - 4}{x + 1} = -8.$$

7. (10 pts.) Evaluate each of the following thorny limits:

(a) $\lim_{x \rightarrow +\infty} \frac{\ln(2x)}{\ln(x^3)} =$

(b) $\lim_{x \rightarrow 0} \frac{(x+25)^{1/2} - 5}{x} =$

8. (15 pts.) (a) (5 pts.) Carefully sketch both $f(x) = \ln(x)$ and $g(x) = e^x$ on the coordinate system below. **Label very carefully.**



(b) (10 pts.) Evaluate each of the following limits.

$$\lim_{x \rightarrow -\infty} e^x =$$

$$\lim_{x \rightarrow \infty} e^x =$$

$$\lim_{x \rightarrow 0^+} \ln(x) =$$

$$\lim_{x \rightarrow \infty} \ln(x) =$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{3x} =$$

Silly 10 point Bonus Problem: Provide an $\varepsilon - \delta$ proof that

$$\lim_{x \rightarrow 2} \frac{1}{x} = \frac{1}{2}.$$

Say where your work is, for it won't fit here.