Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇨" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me all the magic on the page. Eschew obfuscation.*

1. (10 pts.) (a) Using complete sentences and appropriate notation, provide the precise mathematical definition of **continuity** of a function f(x) at a point x = a.

(b) Is there a real number k, that will make the function $f(x)$ defined below continuous at $x = 0$? Either find the value for k and using the definition of continuity, prove that it makes f continuous at $x = 0$, or explain completely why there cannot be such a number k. Suppose

$$f(x) = \begin{cases} \dfrac{\sin(x)}{|x|} & , \ x \neq 0 \\ k & , \ x = 0 \end{cases}$$

2. (10 pts.) (a) Using complete sentences and appropriate notation, state the Intermediate Value Theorem.

(b) Show that the equation $x^3 + x^2 - 2x = 1$ has at least one solution in the interval [-1,1].

3. (5 pts.) Pretend f is a magical function that has the property that at x = 3 the tangent line f is actually defined by the equation y = -2(x - 1) + 5. Obtain

(a) $f(3) =$ (b) $f'(3) =$

4. (25 pts.) Compute the first derivatives of the following functions. You may use any of the rules of differentiation that are at your disposal. Do not attempt to simplify the algebra in your answers.

(a) $f(x) = 4x^6 - 7x^{-12} + 8\tan(x)$

$f'(x) =$

(b) $g(x) = (4x^2 - 2x^{-1})\sec(x)$

$g'(x) =$

(c) $h(t) = \dfrac{5t^{10} + 1}{\sin(t) + 2}$

$h'(t) =$

(d) $y = \cot^5(2\theta + 1)$

$\dfrac{dy}{d\theta} =$

(e) $L(z) = \sin(4z^8) + 4csc(\dfrac{\pi}{6}) - 4\cos(\dfrac{z}{2})$

$\dfrac{dL}{dz}(z) =$

5. (10 pts.) What are the x- and y - intercepts of the tangent line to the graph of $y = 1/x^2$ at the point (2,1/4)?

6. (10 pts.) (a) Find all values in the interval $[0,2\pi]$ at which the graph of f has a horizontal tangent line when $f(x) = x + 2\cdot\cos(x)$.

(b) The following limit represents $f'(a)$ for some function f and some number a. Using that information, evaluate the limit.

$$\lim_{x \to \pi} \frac{\sin(3x) - 0}{x - \pi} =$$

7. (10 pts.) (a) Using complete sentences and appropriate notation, provide the precise mathematical definition for the derivative, $f'(x)$, of a function $f(x)$.

(b) Using only the definition of the derivative as a limit, show all steps of the computation of $f'(x)$ when $f(x) = 1/x$.

$f'(x) =$

8. (5 pts.) Determine whether the following function is differentiable at x = 1.

$$f(x) = \begin{cases} x^2 + x + 2 & , \ x \le 1 \\ 3x & , \ x > 1 \end{cases}$$

9. (5 pts.) Compute f″(x) when

$$f(x) = \sin(2x^3).$$

10. (10 pts.) A spherical balloon is to be deflated so that its radius decreases at a constant rate of 15 cm/min. At what rate must air be removed when the radius is 9 cm.? [$V = (4/3)\pi r^3$??]

Silly 10 point Bonus Problem: Explain completely how to obtain the limit

$$\lim_{x \to 0} \frac{x}{\sin^{-1}(x)} = 1$$

from

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1.$$

Say where your work is, for it won't fit here.