Read Me First: Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me all the magic on the page. Eschew obfuscation.

1. (10 pts.) (a) (6 pts.) Find formulas for Δy and the differential dy when $y = x^2 - 2x + 1$. Label your expressions correctly.

(b) (4 pts.) Use an appropriate local linear approximation formula to estimate $(3.02)^4$.

2. (10 pts.) (a) Use logarithmic differentiation to find dy/dx when

$$y = (x^3 - 2x)^{\ln(x)}$$

(b) Find dy/dx by using implicit differentiation when

$$\cos(xy) = y.$$

3. (10 pts.) (a) Let

$$f(x) = e^{x^3+x}$$

Show that f is one-to-one on $\mathbb{R}$ using f'. [A little explanation is needed!]

(b) Suppose that a one-to-one function f has a tangent line given by $y = 5x + 3$ at the point (1,8). Find $f^{-1}(8)$ and $(f^{-1})'(8)$.

$f^{-1}(8) =$

$(f^{-1})'(8) =$

4. (10 pts.) Write down each of the following derivatives. [2 pts/part.]

(a) $\dfrac{d[\tan^{-1}]}{dx}(x) =$

(b) $\dfrac{d[\sin^{-1}]}{dx}(x) =$

(c) $\dfrac{d[csc^{-1}]}{dx}(x) =$

(d) $\dfrac{d[\cos^{-1}]}{dx}(x) =$

(e) $\dfrac{d[\cot^{-1}]}{dx}(x) =$

5. (10 pts.) Fill in the blanks appropriately. [DEFINITIONS!!!]

(a) A function f has a relative maximum at x_0 if there is an open interval containing x_0 on which ____________________ for each x in both the interval and the domain of f.

(b) A function f is concave down on an interval (a, b) if the derivative of f is ____________________ on (a, b).

(c) A function f is concave up on an interval (a, b) if the derivative of f is ____________________ on (a, b).

(d) A function f is decreasing on an interval (a, b) if ____________________ whenever $a < x_1 < x_2 < b$.

(e) A function f is increasing on an interval (a, b) if ____________________ whenever $a < x_1 < x_2 < b$.

6. (5 pts.) Find the following limit by interpreting the expression as an appropriate derivative.

$$\lim_{w \to 2} \frac{3\sec^{-1}(w) - \pi}{w - 2} =$$

7. (5 pts.) Suppose the side of a square is measured to be 10 inches with a possible error of ± 1/32 inch. Estimate the error in the computed area of the square by using differentials. [You may leave your result as a fraction. You do not have to convert to decimal form.]

8. (8 pts.) Assume f is continuous everywhere. If

$$f'(x) = x(x - 2)^4$$

find all the critical points of f and at each stationary point apply the second derivative test to determine relative extrema, if possible. If the second derivative test fails at a critical point, apply the first derivative test to determine the true state of affairs there.

9. (12 pts.) Evaluate each of the following limits. If a limit fails to exist, say how as specifically as possible.

(a) $\displaystyle \lim_{x \to \pi^+} \frac{\sin(x)}{x - \pi} =$

(b) $\displaystyle \lim_{x \to +\infty} \left(1 - \frac{3}{x}\right)^x =$

(c) $\displaystyle \lim_{x \to 0} \frac{x - \tan(x)}{x^3} =$

10. (20 pts.) When f is defined by

$$f(x) = 2x^4 - 4x^2 = 2x^2(x+\sqrt{2})(x-\sqrt{2}) \quad \mathit{for}\ x \in \mathbb{R},$$

$$f'(x) = 8x^3 - 8x = 8x(x+1)(x-1) \quad \text{and} \quad f''(x) = 24x^2 - 8 = 24\left(x+\sqrt{\tfrac{1}{3}}\right)\left(x-\sqrt{\tfrac{1}{3}}\right).$$

(a) (3 pts.) What are the critical point(s) of f and what is the value of f at each critical point?

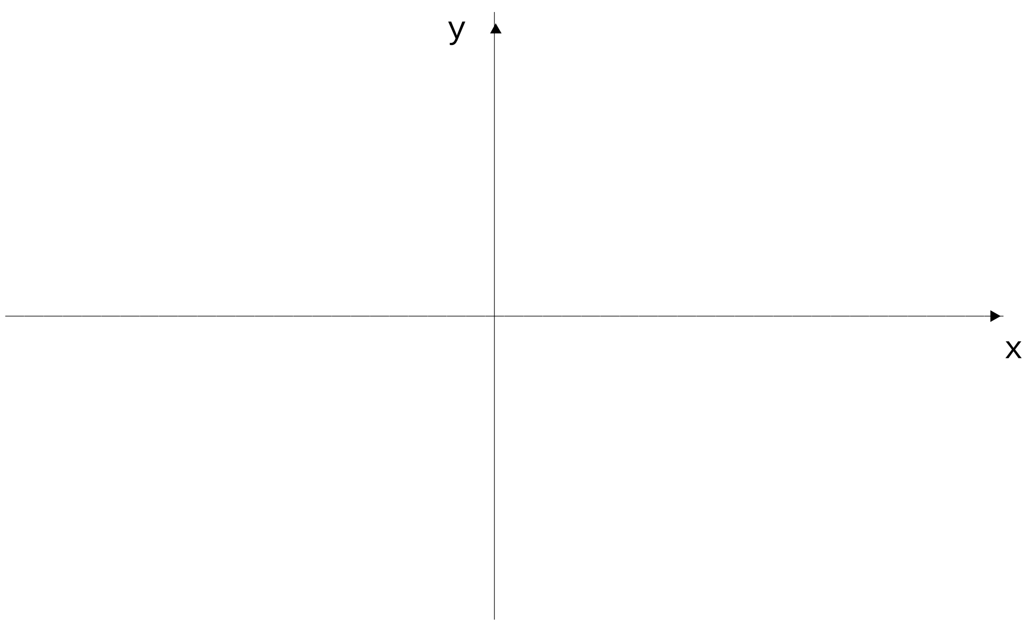
(b) (3 pts.) Determine the open intervals where f is increasing or decreasing.

(c) (3 pts.) Determine the open intervals where f is concave up or concave down.

(d) (3 pts.) List any inflection points or state that there is none.

(e) (3 pts.) Locate any x-intercepts of *f*, or state that there isn't any.

(f) (5 pts.) Carefully sketch the graph of f below by plotting a few essential points and then connecting the dots appropriately.



Silly 10 point Bonus Problem: Show

$$\ln(x+1) \le x \quad \mathit{if} \quad x \ge 0.$$

[Say where your work is, for it won't fit here!]