STUDENT NUMBER: EXAM NUMBER:

Read Me First: *Read each problem carefully and do exactly what is requested. Full credit will be awarded only if you show all your work neatly, and it is correct. Use complete sentences and use notation correctly. Remember that what is illegible or incomprehensible is worthless. Communicate. Eschew obfuscation. Good Luck!*
[Total Points Possible: 120]

1. (10 pts.) Provide each of the following very basic antiderivatives. *Do not forget the arbitrary constant of integration*. 1 point/part.

(a) $\int \frac{1}{|x|\sqrt{x^2-1}}\,dx =$

(b) $\int \sec(x)\tan(x)\,dx =$

(c) $\int \sec^2(x)\,dx =$

(d) $\int \frac{1}{\sqrt{1-x^2}}\,dx =$

(e) $\int \frac{1}{\sqrt{x}}\,dx =$

(f) $\int x^8\,dx =$

(g) $\int 8^x\,dx =$

(h) $\int \frac{1}{x}\,dx =$

(i) $\int \frac{1}{1+x^2}\,dx =$

(j) $\int \cos(x)\,dx =$

2. (5 pts.) Find a function *g(x)* so that *g* satisfies the following equation:

$$\int g(x)\,dx = \sec^2(x) + e^{2x} + 5x^2 + C$$

$g(x) =$

3. (5 pts.) Solve the following initial value problem:

$$\frac{dy}{dt} = \frac{t+1}{\sqrt{t}}, \quad y(1) = 4.$$

4. (8 pts.) *Let* $f(x) = x^3 - 2x^2$ *for* $x \in [1,4]$.

Determine the maximum and minimum values of the function on [1,4] and where they occur. Before you attempt to find ~~them~~ the extrema, explain how you know $f(x)$ actually has both a maximum and a minimum.
[Hint: Pay attention to *the domain* of the function *f* above.]

5. (6 pts.) Use an appropriate local linear approximation formula to estimate $\sqrt{101}$.

6. (6 pts.) Obtain an equation for the line tangent to the graph of the curve defined by

$$2y + \sin(y) = 4x$$

at the point $\left(\frac{\pi}{2}, \pi\right)$, which is actually on the curve.

7. (8 pts.) A rectangular area of 1600 square feet is to be fenced. Three of the sides will use fencing costing $4.00 per running foot, and the remaining side will use fencing costing $1.00 per running foot. Find the dimensions of the rectangle which lead to the least cost to fence the area, and prove that these dimensions actually result in the minimum cost by means of an appropriate analysis.

8. (12 pts.) Evaluate each of the following integrals or anti-derivatives.

(a) $\int 9x^2 - \frac{5}{x} + 3\sec^2(x)\; dx =$

(b) $\int \frac{1}{16+x^2} - \frac{3\ln^2(x)}{x}\; dx =$

(c) $\int \frac{12}{\sqrt{1-4x^2}} + 10xe^{x^2}\; dx =$

9. (12 pts.) Compute the derivatives of the following functions. You may use any of the rules of differentiation that are at your disposal. Do not attempt to simplify the algebra in your answers. 4 pts./part

(a) $f(x) = \sin(3x^2)e^{\pi x}$

$f'(x) =$

(b) $g(x) = \dfrac{\ln(4x)}{x^5}$

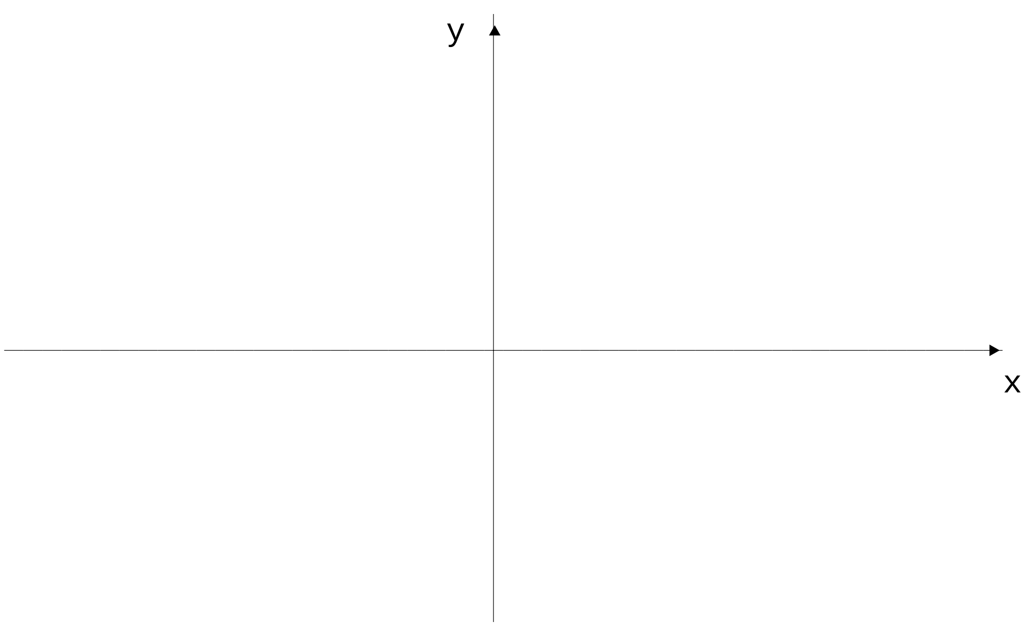
$g'(x) =$

(c) $h(t) = \ln(\tan^{-1}(t^2-1))$

$h'(t) =$

10. (8 pts.) (a) Let $x = \cos(t)$ and $y = \sin^2(t)$ for $t \in [0,\pi]$. Find dy/dx and d^2y/dx^2 at $t = \pi/6$ without eliminating the parameter.

(b) Sketch the curve given in (a) above by eliminating the parameter, and indicate the direction of increasing t. [Pythagoras rocks??]



11. (8 pts.) (a) Using a couple of sentences, provide the precise mathematical definition of $\lim_{x \to a} f(x) = L$ in terms of epsilons and deltas.

(b) Using only the definition of limit at a point in terms of epsilons and deltas, give a proof that $\lim_{x \to 1} \frac{3x^2 - 3}{x - 1} = 6$.

12. (8 pts.) (a) State the Mean-Value Theorem of Differential Calculus. Use a complete sentence and appropriate notation.

(b) By applying the Mean-Value Theorem to the function $f(x) = \ln(x)$ on the interval [1,2] and making a key observation about the derivative of f, prove $0.5 < \ln(2) < 1$.

13. (8 pts.) (a) Use logarithmic differentiation to differentiate

$$f(x) = x^x = e^{x \ln(x)} \quad for\ x > 0.$$

(b) Reveal the essential details needed to see why $\lim_{x \to 0^+} x^x = 1$.

14. (8 pts.) Evaluate each of the following limits.

(a) $\lim_{x \to 0} \frac{8 - 8\cos(x)}{e^{x} + e^{-x} - 2} =$

(b) $\lim_{x \to \infty} x\left[\sec\left(\frac{\pi}{3} + \frac{1}{x}\right) - 2\right] =$

15. (8 pts.) When *f* is defined by

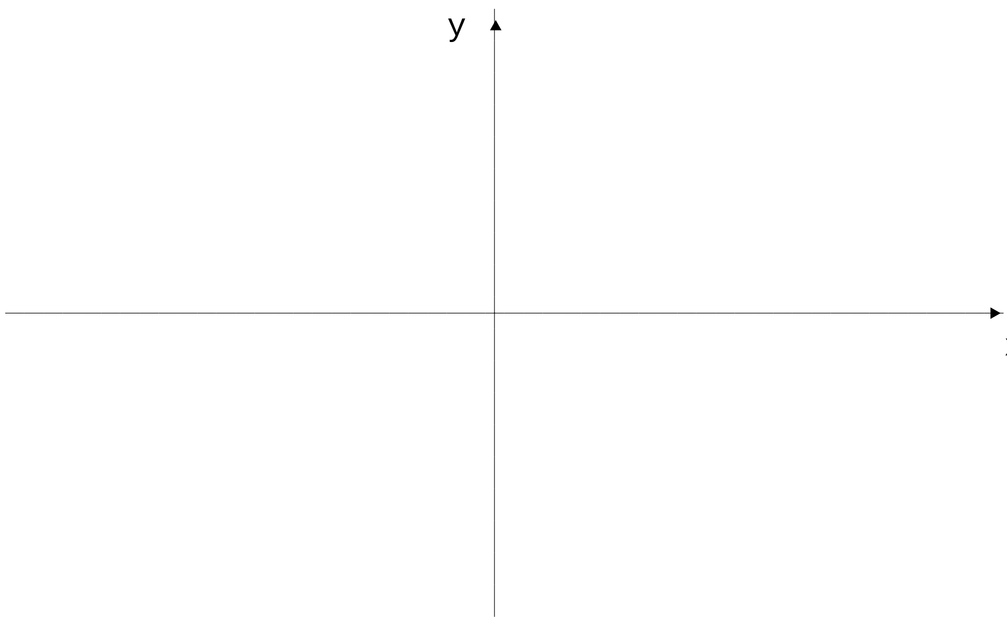
$$f(x) = \frac{2x}{x^2 + 1}$$

on the real line, it turns out that

$$f'(x) = \frac{-2(x-1)(x+1)}{(x^2+1)^2} \quad \text{and} \quad f''(x) = \frac{4x(x-\sqrt{3})(x+\sqrt{3})}{(x^2+1)^3}.$$

On the back of Page 5 of 6, analyze the behavior of *f* with enough completeness so that you can sketch its graph below correctly.

The points to plot are $(-\sqrt{3}, -\frac{\sqrt{3}}{2})$, $(-1,-1)$, $(0,0)$, $(1,1)$, *and* $(\sqrt{3}, \frac{\sqrt{3}}{2})$.



Bonkers 10 Point Bonus: You may attempt at most one of the following:

(A) Show how to use the Mean-Value Theorem to prove the following result: If f is continuous on a closed interval [a,b] with $f'(x) < 0$ for each x in (a,b), then f is decreasing on the closed interval [a,b].

(B) By using the Mean-Value Theorem, prove the error in the value provided by the local linear approximation of $(101)^{1/2}$ in Problem 5 on Page 2 of 6 is less than $(1/2)10^{-3}$. This is three decimal place accuracy.