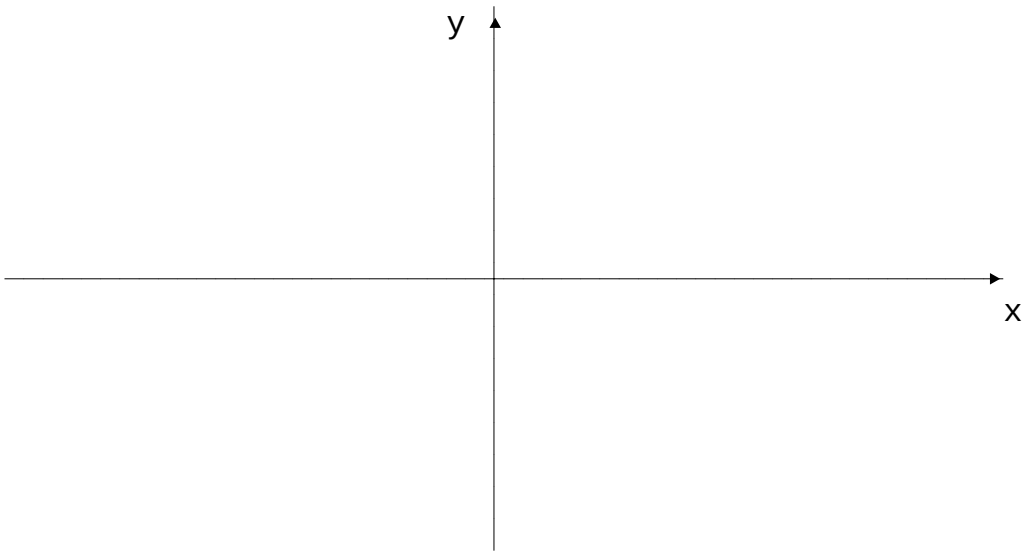
Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇨" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.*

1. (20 pts.) Let

$$g(x) = \begin{cases} x-1 \text{ , if } x \le -1 \text{ or } 1 \le x \\ 1-x^2 \text{ , if } -1 < x < 1 \end{cases}$$

(a) (8 pts.) On the coordinate system below sketch the graph of the function g defined above. Label very carefully.



(b) (12 pts.) Evaluate each of the following easy limits concerning the function g above.

(i) $\lim_{x \to -\infty} g(x) =$ (ii) $\lim_{x \to -1^-} g(x) =$ (iii) $\lim_{x \to -1^+} g(x) =$

(iv) $\lim_{x \to 1} g(x) =$ (v) $\lim_{x \to +\infty} g(x) =$

(vi) $\lim_{x \to 0} \frac{g(x)-g(0)}{x-0} =$

2. (5 pts.) Express the following function in piecewise defined form without using absolute values:

$f(x) = 5|x-2| - 4x =$

3. (25 pts.) For each of the following, find the limit if the limit exists. If the limit fails to exist, say so. Be as precise as possible here. [Work on the back of Page 2 of 4 if you run out of room here.]

(a) $\lim_{x \to 3} \frac{x^2-4}{x+2}$ =

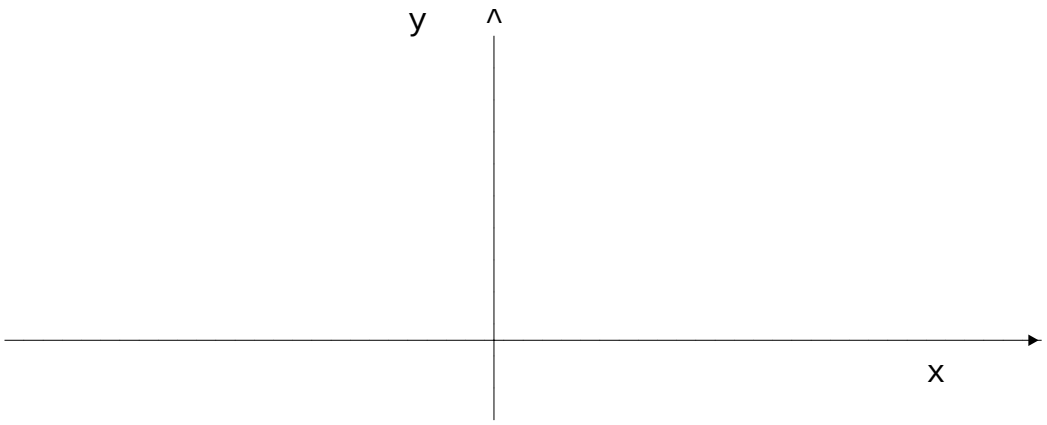
(b) $\lim_{x \to -2} \frac{x^2-4}{x+2}$ =

(c) $\lim_{x \to 2^-} \frac{x+2}{x^2-4}$ =

(d) $\lim_{x \to 1^-} \frac{5x-5}{|x-1|}$ =

(e) $\lim_{x \to -\infty} \frac{\sqrt{25x^2-2}}{2x+3}$ =

4. (7 pts.) Carefully sketch $f(x) = \cot^{-1}(x)$ on the coordinate system below. **Label very carefully**.



Then evaluate the following two limits:

$\lim_{x \to \infty} \cot^{-1}(x) =$ $\lim_{x \to -\infty} \cot^{-1}(x) =$

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5. (8 pts.) If $f(x) = 3x^2$ and $h \neq 0$, then simplifying as much as possible allows us to write

$$\frac{f(x+h) - f(x)}{h} =$$

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6. (10 pts.) (a) Using complete sentences and appropriate notation, provide the precise mathematical definition for

$$\lim_{x \to a} f(x) = L.$$

(b) Using the ε - δ definition of limit, provide a complete proof that

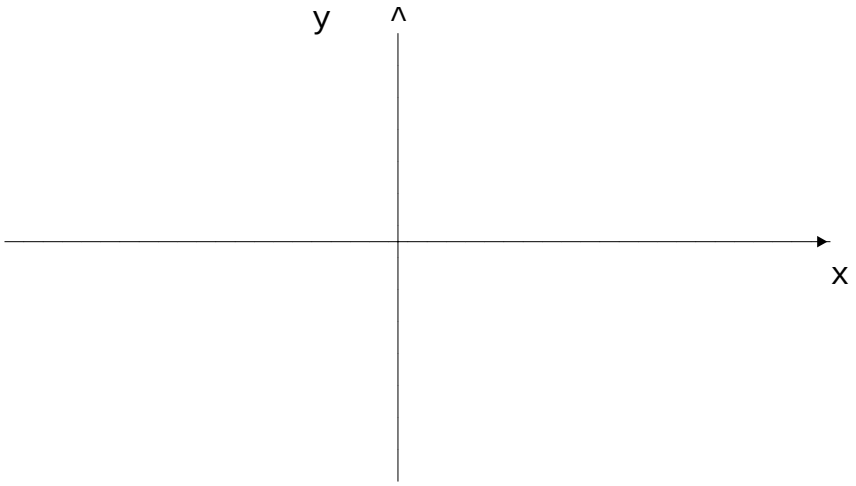
$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = 6.$$

7. (10 pts.) Evaluate each of the following thorny limits:

(a) $\lim_{x \to +\infty} \left(\sqrt{x^2 + 2ax} - x \right) =$

(b) $\lim_{x \to 0} \dfrac{\sqrt{x+16} - 4}{x} =$

8. (15 pts.) (a) (5 pts.) Carefully sketch both f(x) = ln(x) and g(x) = e^x on the coordinate system below. **Label very carefully**.



(b) (10 pts.) Evaluate each of the following limits.

$\lim_{x \to 0^+} \ln(x) =$

$\lim_{x \to \infty} \ln(x) =$

$\lim_{x \to -\infty} e^x =$

$\lim_{x \to \infty} e^x =$

$\lim_{x \to \infty} \left(1 + \dfrac{1}{x} \right)^{4x} =$

Silly 10 point Bonus Problem: Provide an ε - δ proof that

$$\lim_{x \to 25} \sqrt{x} = 5.$$

Say where your work is, for it won't fit here.