

---

Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Communicate. Show me all the magic on the page. Eschew obfuscation.*

---

---

7. (10 pts.) Find the x-coordinates of all points on the graph of  $y = 3 - x^2$  at which the tangent line passes through the point (2,0).

Here is a second solution to this problem. This is based on a partially correct solution provided by a student.

Instead of first obtaining an equation for the tangent line to the curve at an arbitrary point  $x_0$  on the real line, we will obtain an equation for the line through the point (2,0) with slope provided by that of the tangent line at any point  $(x_0, f(x_0))$  on the curve. Plainly, this is given by

$$y - (0) = -2x_0(x-2) ,$$

or in slope-intercept form,

$$y = -2x_0x + 4x_0 ,$$

after routine algebra. Since the line defined by this equation also contains the point  $(x_0, f(x_0))$ , we must have

$$3 - x_0^2 = -2x_0x_0 + 4x_0 .$$

Consequently, as in the first solution,

$$\begin{aligned} 0 &= x_0^2 - 4x_0 + 3 \\ &= (x_0 - 1)(x_0 - 3) . \end{aligned}$$

Thus,  $x_0 = 1$  or  $x_0 = 3$ .