NAME:

Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Communicate. Show me all the magic on the page. Eschew obfuscation.*

---

1. (25 pts.) Compute the first derivatives of the following functions. You may use any of the rules of differentiation that are at your disposal. Do not attempt to simplify the algebra in your answers. You should do minor arithmetic, however, to have clean constants.

(a) $f(x) = 6x^4 - 12x^{-7} + 8\cot(x)$

$f'(x) =$

(b) $g(x) = (2x^2 - 4x^{-1})\csc(x)$

$g'(x) =$

(c) $h(t) = \dfrac{10t^5 + 1}{2 - \cos(t)}$

$h'(t) =$

(d) $y = \tan^5(2\theta + 1)$

$\dfrac{dy}{d\theta} =$

(e) $L(z) = \sqrt{4z^8} + 4\sec(\frac{\pi}{6}) - 4\sin(\frac{z}{8})$

$\dfrac{dL}{dz}(z) =$

2. (10 pts.) (a) Using complete sentences and appropriate notation, provide the precise mathematical definition of **continuity** of a function f(x) at a point x = a.

(b) Is there a real number *k*, that will make the function *f*(*x*) defined below continuous at $x = \cancel{\pi/4}$ $\pi/3$? (Corrected in class.) Either find the value for *k*, or explain completely why there cannot be such a number *k*. Suppose

$$f(x) = \begin{cases} \dfrac{\cos(x) - (1/2)}{x - (\pi/3)} & , \ x \neq \pi/3 \\ k & , \ x = \pi/3 \end{cases}$$

3. (5 pts.) Suppose that the line defined by $2x + 3y = 11$ is tangent to the graph of $y = f(x)$ at $x = 1$. Then

$f(1) =$ and $f'(1) =$

4. (10 pts.) (a) Using complete sentences and appropriate notation, provide the precise mathematical definition for the derivative, *f'*(*x*), of a function *f*(*x*).

(b) *Using only the definition of the derivative as a limit*, show all steps of the computation of $f'(x)$ when $f(x) = x^3$

$f'(x) =$

5. (10 pts.) (a) Using complete sentences and appropriate notation, state the Intermediate-Value Theorem.

(b) Using the Intermediate-Value Theorem, show that the equation

$x^3 - x - 1 = 0$ has at least one solution in the interval [1,2].

6. (10 pts.) A softball diamond is a square whose sides are 60 feet long. Suppose that a player running from first to second base has a speed of 25 feet per second at the instant when she is 10 feet from second base. At what rate is the player's distance from home plate changing at that instant?

7. (10 pts.) Find the x-coordinates of all points on the graph of

$y = 3 - x^2$ at which the tangent line passes through the point (2,0).

8. (5 pts.) $$\lim_{x \to 0} 2x^2 \sin(\ln|x|) = 0$$

The limit above cannot be obtained using the arithmetic of limits. How does one legitimately see the truth of this limit easily??

9. (5 pts.) Compute $f''(x)$ when $f(x) = x\tan(x)$.

10. (10 pts.) Suppose that *f* and *g* are two differentiable functions with

$$f(2) = 3, \quad f'(2) = -4, \textit{ and } \; f'(-4) = -8,$$

$$\textit{and}$$

$$g(2) = -4, \quad g'(2) = 5, \textit{ and } \; g'(3) = 7.$$

Using only the information above and appropriate differentiation rules, *either* compute the exact value of *h'*(2), when *h* is as defined in terms of *f* and *g* in each part *or* if there is essential data missing, say what it is.

(a) If $h(x) = 8f(x) + 25$, then $h'(2) =$

(b) If $h(x) = f(x)g(x)$, then $h'(2) =$

(c) If $h(x) = \dfrac{f(x)}{g(x)}$, then $h'(2) =$

(d) If $h(x) = f(g(x))$, then $h'(2) =$

(e) If $h(x) = g(f(x))$, then $h'(2) =$

10 point Bonus: Provide a rigorous ε-δ proof of the limit in Problem 8, above. Say where your work is, for it won't fit here.