Read Me First: Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me all the magic on the page. Eschew obfuscation.

1. (10 pts.) Write down each of the following derivatives. [2 pts/part.]

(a) $\frac{d[\tan^{-1}]}{dx}(x) =$

(b) $\frac{d[\sin^{-1}]}{dx}(x) =$

(c) $\frac{d[\sec^{-1}]}{dx}(x) =$

(d) $\frac{d[\cos^{-1}]}{dx}(x) =$

(e) $\frac{d[\cot^{-1}]}{dx}(x) =$

2. (10 pts.) (a) Use logarithmic differentiation to find *dy/dx* when

$$y = x^{\tan(x)}$$

(b) Find *dy/dx* when $\sin(xy) = y$.

3. (10 pts.) (a) (6 pts.) Find formulas for Δy and the differential dy when $y = x^2$. Label your expressions correctly.

(b) (4 pts.) Use an appropriate local linear approximation formula to estimate $\sqrt{24.9}$.

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4. (10 pts.) (a) Let

$$f(x) = \frac{\exp(4 - x^2)}{x} \quad for \quad x > 0 .$$

Show that f is one-to-one on $(0,\infty)$ using f'. [A little explanation is needed!]

(b) Let $g(x) = f^{-1}(x)$, where f is from part (a) of this problem.

Compute $g'\left(\frac{1}{2}\right)$. Hint: $f(2)$ = ??

5. (10 pts.) Fill in the blanks appropriately. [*DEFINITIONS!!!*]

(a) A function f has a relative minimum at x_0 if there is an open interval containing x_0 on which ____________________ for each x in both the interval and the domain of f.

(b) A function f is increasing on an interval (a, b) if ____________________ whenever $a < x_1 < x_2 < b$.

(c) A function f is concave down on an interval (a, b) if the derivative of f is ____________________ on (a, b).

(d) A function f is decreasing on an interval (a, b) if ____________________ whenever $a < x_1 < x_2 < b$.

(e) A function f is concave up on an interval (a, b) if the derivative of f is ____________________ on (a, b).

6. (5 pts.) Suppose the side of a cube is measured to be 10 inches with a possible error of ± 1/10 inch. Estimate the error in the computed volume of the cube by using differentials. [You may leave your result as a fraction. You do not have to convert to decimal form.]

7. (5 pts.) Find the following limit by interpreting the expression as an appropriate derivative. Complete the sentence outlined below appropriately.

(*) $\lim_{x \to 2} \frac{10^x - 100}{x - 2} =$

since the limit (*) is actually $f'(a)$, where $f(x) =$

and $a =$

8. (8 pts.) Assume f is continuous everywhere. If

$$f'(x) = (x - 1)(x - 2)^3 ,$$

find all the critical points of f and at each stationary point apply the second derivative test to determine relative extrema, if possible. If the second derivative test fails at a critical point, say as much and then apply the first derivative test to determine the true state of affairs.

9. (12 pts.) Evaluate each of the following limits. If a limit fails to exist, say how as specifically as possible.

(a) $\displaystyle\lim_{x \to 0} \frac{\sin^{-1}(3x)}{x} =$

(b) $\displaystyle\lim_{x \to 0^+} (\sin(x))^{4/\ln(x)} =$

(c) $\displaystyle\lim_{x \to +\infty} (\ln(e^{x+5} + 3) - x) =$

10. (20 pts.) When f is defined by

$$f(x) = 4x^3 - 3x^4 = -3x^3\left(x - \frac{4}{3}\right) \quad for\ x \in \mathbb{R},$$

$$f'(x) = 12x^2 - 12x^3 = -12x^2(x-1) \quad \text{and} \quad f''(x) = 24x - 36x^2 = -36x\left(x - \frac{2}{3}\right).$$

(a) (3 pts.) What are the critical point(s) of f and what is the value of f at each critical point?

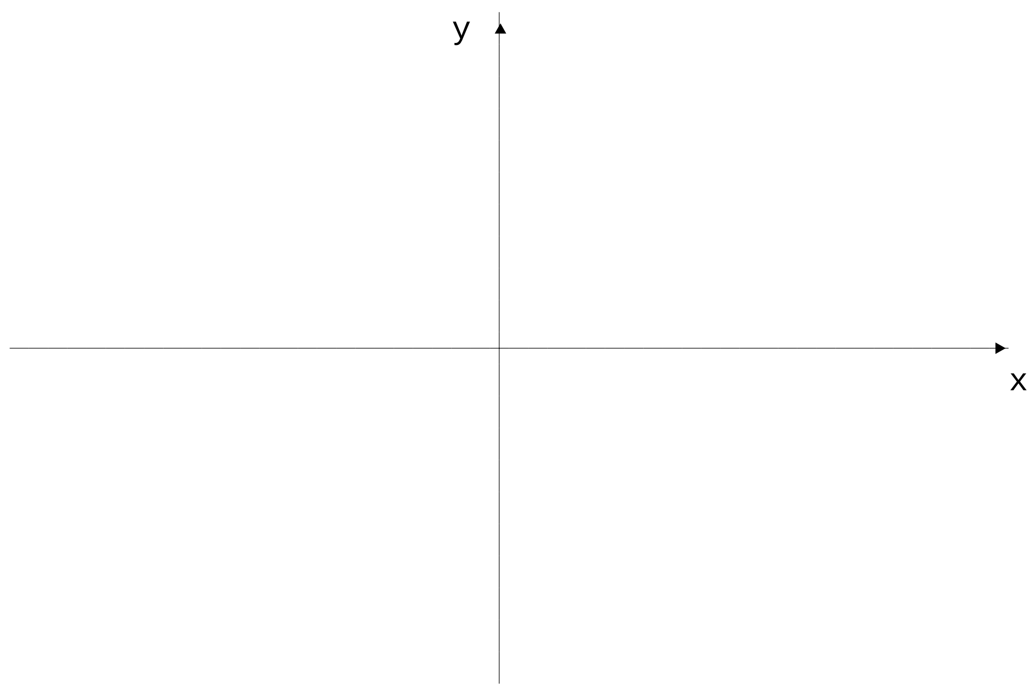
(b) (3 pts.) Determine the open intervals where f is increasing or decreasing.

(c) (3 pts.) Determine the open intervals where f is concave up or concave down.

(d) (3 pts.) List any inflection points or state that there is none.

(e) (3 pts.) Locate any x-intercepts of f, or state that there isn't any.

(f) (5 pts.) Carefully sketch the graph of f below by plotting a few essential points and then connecting the dots appropriately.



Silly 10 point Bonus Problem: Show

$$\sqrt[3]{1+x} < 1 + \frac{1}{3}x \quad if \quad x > 0.$$

[Say where your work is, for it won't fit here!]