NAME:

STUDENT NUMBER: **EXAM NUMBER:**

READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Since the answer really consists of all the magic transformations and incantations, do not "box" your final results. Show me all the magic on the page.*

1. (a)(10 pts.) Find both the Maclaurin polynomial, $P_3(x)$, and the Lagrange form of the remainder term, $R_3(x)$, for the function $f(x) = \cos(x)$. Then write $\cos(x)$ in terms of $P_3(x)$ and $R_3(x)$.

$P_3(x) =$

$R_3(x) =$

$\cos(x) =$

(b) (5 pts.) Provide a reasonable estimate of how many decimal places accuracy your computation would provide if you used $P_3(x)$ to approximate $\cos(x)$ when $|x| < 1/10$.

2. (10 pts.) (a) Using literal constants A, B, C, etc., write the form of the partial fraction decomposition for the proper fraction below. Do not attempt to obtain the actual numerical values of the constants A, B, C, etc.

$$\frac{x^2+25}{x^3(x-2)(x^2+1)} =$$

(b) Now obtain the indefinite integral of the rational function of part (a) in terms of the constants A, B, C, etc. using the partial fraction decomposition. Do not attempt to obtain the numerical values of the constants.

$$\int \frac{x^2+25}{x^3(x-2)(x^2+1)}\,dx =$$

3. (15 pts.) Suppose that

$$f(x) = \sum_{n=1}^{\infty} \frac{1}{n5^n}(x-2)^n$$

(a) (5 pts.) Find the radius of convergence and the interval of convergence of the power series function f.

(b) (5 pts.) By using sigma notation and doing termwise differentiation, obtain a power series for f′(x). What is the radius of convergence of the series for f′(x)?

f′(x) =

(c) (5 pts.) Obtain an infinite series whose sum is the same as the numerical value of the following definite integral. [We are working with the function f of part (a) of this problem. Integrate termwise. **Use sigma notation**.]

$$\int_2^4 f(x)dx =$$

4. (10 pts.) (a) Find the rational number represented by the following repeating decimal.

0.366366366... =

(b) Find all values of x for which the given geometric series converges, and then express the closed form sum of the series as a function of x.

$$\sum_{n=1}^{\infty}\left(\frac{(x-5)}{10}\right)^n =$$

5. (25 pts.) Find each of the following antiderivatives.

(a) $\int \frac{x^3 + 4x}{x^2 + 1}\, dx =$

(b) $\int (1 - x^2)^{1/2}\, dx =$

(c) $\int 2x \cos(x)\, dx =$

(d) $\int \ln(x^2 + 1) 2x\, dx =$

(e) $\int \frac{4}{x(x^2 + 1)}\, dx =$

6. (25 pts.) (a) (7 pts.) State the Fundamental Theorem of Calculus.

(b) (6 pts.) Compute g′(x) when g(x) is defined by the following equation.

$$g(x) = \int_0^x \sec^3(t)\,dt + \tan(x)$$

g′(x) =

(c) (6 pts.) Write the solution to the following initial value problem in terms of a definite integral with respect to the variable t, but don't attempt to evaluate the definite integral:

$$y'(x) = e^{\tan(x)}, \quad y(\pi/3) = 8.$$

y(x) =

(d) (6 pts.) Suppose that f(1/2) = 0 and

$$f'(x) = \frac{2}{1 + 4x^2}\ .$$

What function f(x) satisfies these two equations?? [Identify f as completely as possible. f can be written in terms of an old friend.]

f(x) =

7. (25 pts.) Obtain the exact numerical value of each of the following if possible. If a limit doesn't exist or an improper integral or an infinite series fails to converge, say so as precisely as possible. [Warning: If you evaluate improper integrals improperly, you will lose most of the points possible on the problem part. Pay attention to details.]

(a) $\lim_{n \to \infty} \frac{1}{n}\sum_{k=1}^{n} \cos\left(\frac{\pi}{2}\left(\frac{k}{n}\right)\right) =$

(b) $\int_{1}^{\infty} \frac{10}{x^2}\, dx =$

(c) $\int_{2}^{\infty} \frac{50}{x^2 + x}\, dx =$

(d) $\sum_{n=2}^{\infty} \frac{50}{n^2 + n} =$

(e) $\int_{-1}^{0} \frac{50}{x^2 + 1}\, dx =$

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8. (10 pts.) Using sigma notation, write the Maclaurin series of each of the following functions, and give the interval I where the series converges to the function.

$\sin(x)$ =

e^x =

$\tan^{-1}(x)$ =

$\cos(x)$ =

$\ln(1+x)$ =

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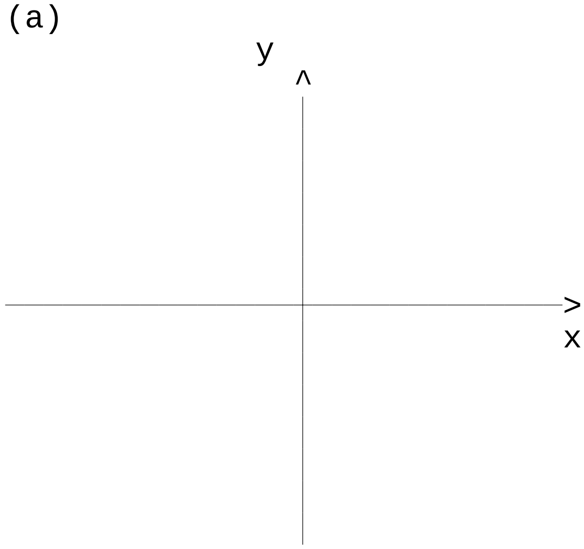
9. (15 pts.) (a) Sketch the region enclosed by the curves $y = \cos(x)$ and $y = 0$ between $x = -\pi/2$ and $x = \pi/2$. Label carefully. (b) If the region is revolved around the x-axis, a solid is formed. Using the method of disks and washers, write down a single definite integral with respect to the variable x which would be used to compute the volume of the solid. **Don't attempt to evaluate the integral**. (c) Using the method of cylindrical shells, write down the definite integral with respect to the variable y that would be used to compute the same volume of the solid of part(b). **Don't attempt to evaluate the integral**.

(a)



(b)

V =

(c)

V =

10. (10 pts) Obtain the arc length along the curve defined by the equation $y = (4 - x^2)^{1/2}$ from $x = (2)^{1/2}$ to $x = (3)^{1/2}$.

11. (15 pts.) Classify each of the following series as absolutely convergent, conditionally convergent, or divergent.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{\sqrt{n}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{3/2}}$

12. (10 pts.) Suppose that $n \geq 2$ is a positive integer. Show in detail how to derive the following reduction formula:

$$\int \cos^{n}(x)\,dx = \frac{\cos^{n-1}(x)\sin(x)}{n} + \frac{n-1}{n}\int \cos^{n-2}(x)\,dx$$

[A parting trig-or-treat?? Thank Pythagoras.]

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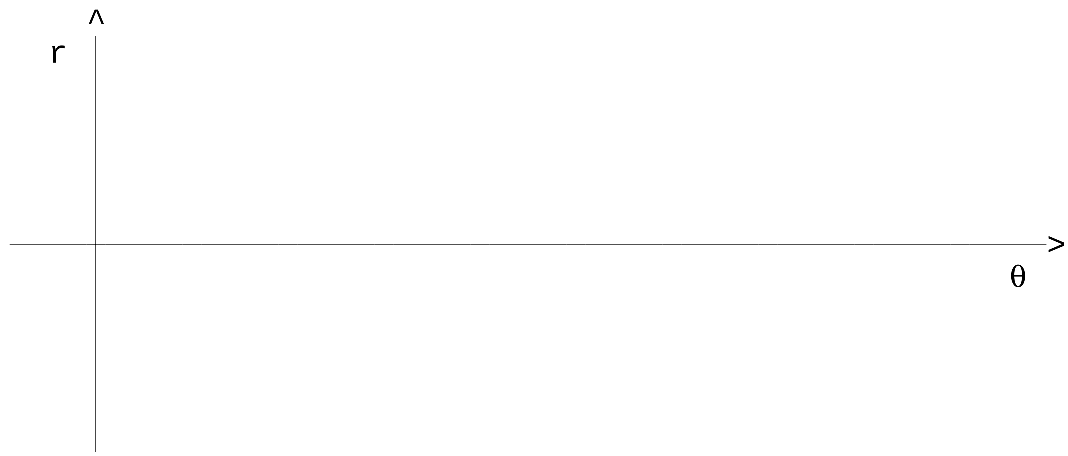
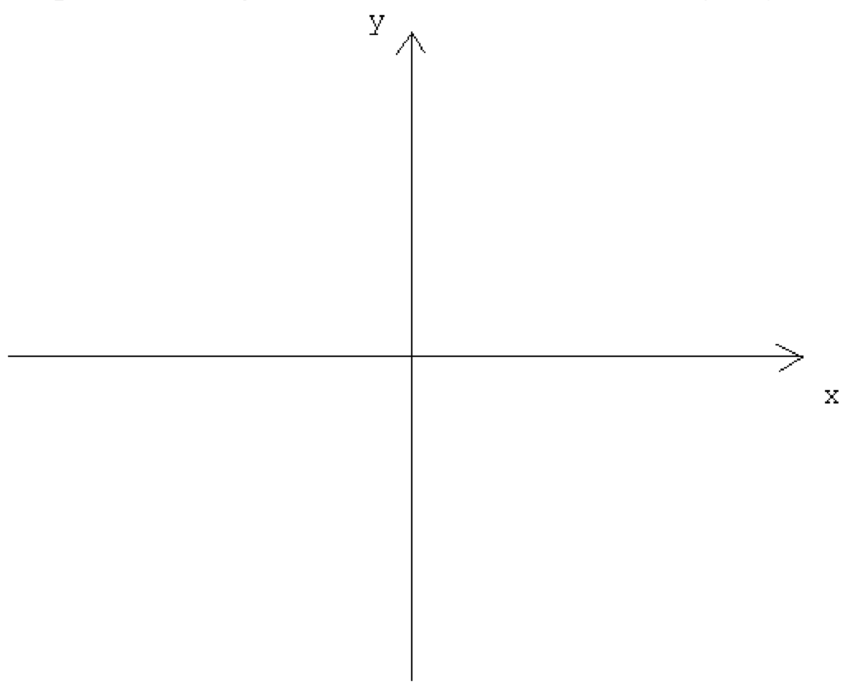
(15 pts.) Sketch the $r = 2 \cdot \sin(2\theta)$ in polar coordinates, and then compute the area of the region enclosed by one loop of the curve. Do this as follows: (a) Carefully sketch the auxiliary curve, a rectangular graph, on the coordinate system provided. (b) Then translate this graph to the polar one. Finally, (c) setup the required polar integral and evaluate it.

(a)



(b) [Think of polar coordinates overlaying the x,y axes below.]



(c)

Area =

Silly 20 point bonus: In Problem 3(c) on Page 2 above you obtained an infinite series whose sum is exactly equal to a definite integral. Now obtain the exact value of that infinite series in terms of your friendly natural log function. [Work on the back of page 1, which faces page 2 where the varmints are. **Hint:** *It helps to figure out who f(x) really is.*]