READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page, for I do not read minds.*

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1. (5 pts.) Complete the equation to write the following sum using sigma notation:

$$\frac{x^1}{2} - \frac{x^2}{4} + \frac{x^3}{6} - \frac{x^4}{8} + \ldots - \frac{x^{22}}{44} =$$

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2. (5 pts.) Find the average value of f(x) = sin(x) over the interval [π/3,π].

f_{AVE} =

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3. (10 pts.) Differentiate the following functions:

(a) $g(x) = \int_x^0 \frac{1}{(1-t^2)^{1/2}}\, dt$ [Note: The domain of g is (-1,1).]

(b) $f(x) = \int_1^{e^x} \frac{1}{1+t^2}\, dt$

Make sure you label your derivatives correctly.

4. (5 pts.) Using appropriate properties of the definite integral and suitable area formulas from geometry, evaluate the following definite integral. [Roughly sketching a couple of simple graphs might help. First use linearity, though.]

$$\int_0^3 2(9 - x^2)^{1/2} - x \, dx =$$

5. (5 pts.) State the Fundamental Theorem of Calculus.

6. (5 pts.) Express the solution to the following initial value problem by using a definite integral with respect to the variable t,

$$\frac{dy}{dx} = (1 + e^x)^{1/2} , \quad y(1) = 10.$$

To do this, fill in the right side of the equation below correctly.

y(x) =

7. (5 pts.) Using only the second comparison property of integrals, give both a lower and an upper bound on the true numerical value of the integral below.

$$I = \int_0^{\pi/6} \cos^2(x) \, dx$$

8. (10 pts.) Assume that $[x_{i-1},x_i]$ denotes the *ith* subinterval of a partition of the interval [0,1] into n subintervals, all with the same length $\Delta x = 1/n$.

(a) Write the value of the following limit as a definite integral.

$$L = \lim_{n \to \infty} \sum_{i=1}^{n} e^{2x_i} \Delta x$$

L =

(b) By evaluating the integral you obtained in part (a) above using the Fundamental Theorem of Calculus, give the exact numerical value of the limit, L.

L =

9. (10 pts.) Reveal all the details of evaluating the given integral by computing

$$\lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x$$

where the sum is assumed to have originated from a regular partition of the given interval of integration. Here, you are to actually compute the Riemann sum in closed form and then evaluate the limit.

$$\int_0^2 x^2 \, dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x =$$

10. (5 pts.) Give an example of a bounded function defined on the closed interval [0,1] that is not Riemann integrable.

11. (5 pts.) Are there any problems with the string of equations used in the computation

$$\int_0^{\pi/4} \tan^2(x)\sec^2(x)\,dx = \int_0^{\pi/4} u^2\,du = \left(\frac{1}{3}u^3\right)\Big|_0^{\pi/4} = \left(\frac{1}{3}\tan^3(x)\right)\Big|_0^{\pi/4} = \frac{1}{3}$$

when we use the substitution u = tan(x) so that du = $\sec^2$(x)dx?? Explain briefly.

12. (10 pts.) (a) Locate the critical points of the function g defined on the interval [0,4π] by means of the equation

$$g(x) = \int_0^x t\sin(t)\,dt.$$

[Note: The critical points are actually in the open interval (0,4π).]

(b) Determine the open intervals in (0,4π) where g is increasing or decreasing.

13. (10 pts.) Evaluate each of the following sums in closed form.

(a) $\sum_{i=0}^{199} \left(\frac{1}{2}\right)^i =$

(b) $\sum_{i=1}^{200} (2i + 1) =$

14. (10 pts.) Evaluate the following definite integral.

$\int_0^2 | x^2 - 1 | \, dx =$

Silly 10 Point Bonus: Theorem 1 of Section 5.6 is called the Average Value Theorem, and its statement follows:

If f is continuous on [a,b], then

$$f(\bar{x}) = \frac{1}{b-a}\int_a^b f(x)\, dx$$

for some $\bar{x}$ in [a,b].

Provide a proof of this by briefly explaining how the conclusion follows from two very important properties of continuous functions via the integral comparison properties.
[Several sentences are needed. Work on the back of Page 4 of 5.]