READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.*

Silly 10 Point Bonus: Consider the following equation:

$$\tan^{-1}\left(\frac{x}{(1-x^2)^{1/2}}\right) = \int_0^x \frac{1}{(1-t^2)^{1/2}}\,dt$$

Is this equation true for each x in the open interval (-1,1)?? On the back of Page 4 of 5, either provide me proof that the equation is true for every x in (-1,1), or show me that there is some number x_0 in the interval where the equation fails to be true.

As you might have guessed, this really is true. A not too horribly difficult way to see this is as follows:

Set

$$f(x) = \tan^{-1}\left(\frac{x}{(1-x^2)^{1/2}}\right) \text{ for } x \in (-1,1),$$

and

$$g(x) = \int_0^x \frac{1}{(1-t^2)^{1/2}}\,dt \text{ for } x \in (-1,1).$$

The question now boils down to whether f and g are the same function. First, both are plainly differentiable and thus, continuous on their domains with

$$f'(x) = \frac{1}{1+\left(\frac{x}{(1-x^2)^{1/2}}\right)^2}\cdot\frac{(1-x^2)^{1/2} - x(1/2)(1-x^2)^{-1/2}(-2x)}{1-x^2}$$

$$= \frac{1-x^2}{1}\cdot\frac{1-x^2+x^2}{(1-x^2)^{3/2}}$$

$$= \frac{1}{(1-x)^{1/2}},$$

and

$$g'(x) = \frac{1}{(1-x^2)^{1/2}} \text{ for } x \in (-1,1).$$

Thus both derivatives are the same on (-1,1). It follows that the two functions differ by a constant. Since f(0) = 0 = g(0), the two functions are the same on (-1,1), and so the equation is true on the same interval.//