READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.*

1. (30 pts.) Here are five trivial trigonometric integrals to evaluate. [6 pts./part]

(a) $\int \tan^2(x)\,dx =$

(b) $\int \cot(4x)\,dx =$

(c) $\int \frac{\sin^2(t)}{\cos(t)}\,dt =$

(d) $\int \cos(x)\cos(4x)\,dx =$

(e) $\int \tan(t)\sec^4(t)\,dt =$

2. (20 pts.) Evaluate each of the following antiderivatives [5 pts./part]

(a) If x > 1, then

$$\int \sec^{-1}(x)\,dx =$$

(b) $\displaystyle\int \frac{1}{x^2-2x+26}\,dx =$

(c) $\displaystyle\int (4 - t^2)^{1/2}\,dt =$

(d) $\displaystyle\int \frac{x^4}{x^3+x}\,dx =$

3. (10 pts.) (a) Using literal constants A, B, C, etc., write the form of the partial fraction decomposition for the proper fraction below. Do not attempt to obtain the actual numerical values of the constants A, B, C, etc.

$$\frac{x^2+4}{x(x-1)^3(x^2+1)} =$$

(b) Now obtain the indefinite integral of the rational function of part (a) in terms of the constants A, B, C, etc. using the partial fraction decomposition. Do not attempt to obtain the numerical values of the constants.

$$\int \frac{x^2+4}{x(x-1)^3(x^2+1)}\,dx =$$

4. (5 pts.) Evaluate the following, quite proper, definite integral:

$$\int_{-1}^{1} \frac{1}{(1+x^2)^{1/2}}\,dx =$$

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5. (5 pts.) Find a pattern in the sequence with given terms a_1, a_2, a_3, a_4 , and assuming that it continues as indicated, write a formula for the general term a_n of the sequence.

-1/2, +3/5, -5/8, +7/11, ...

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6. (5 pts.) Assume that the sequence $\{x_n\}$ is defined recursively by the formula

$$x_{n+1} = (5 + x_n)^{1/2} \text{ if } n \geq 1.$$

If $x_1 = 1$ and if $L = \lim_{n \to \infty} x_n$ exists, what is the exact value of L??

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7. (10 pts.) Determine whether the sequence $\{a_n\}$ converges, and find its limit if it does.

(a) $a_n = \int_1^n \frac{1}{x^2 + x} dx$

(b) $a_n = n \cdot \sin(\frac{2\pi}{n})$

8. (15 pts.) Evaluate each of the following integrals. Look before you leap.

(a)

$\int_0^{\infty} e^{-5x}\, dx =$

(b)

$\int_0^{10} (10 - x)^{-1}\, dx =$

(c)

$\int_0^{1} \ln(x)\, dx =$

Silly 10 Point Bonus: The sequence defined recursively in Problem 6 above really does converge. Prove it does by establishing that the sequence is bounded, with an explicit appropriate bound, and monotone, in a suitable sense. Yes, an induction argument or two is in the offing. Tell me where your work is, for there isn't room here.