READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.*

---

10. (15 pts.) (b) Find a power series representation for the function f(x) below by doing termwise integration. Write your answer using sigma notation.

$$f(x) = \frac{\pi}{2} - \int_0^x \frac{1}{1+t^2}\,dt = \frac{\pi}{2} - \int_0^x \sum_{k=0}^{\infty} (-1)^k t^{2k}\,dt$$

$$= \frac{\pi}{2} - \sum_{k=0}^{\infty} (-1)^k \int_0^x t^{2k}\,dt = \frac{\pi}{2} - \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1}$$

for -1 < x < 1.

---

10 Point Bonus: (a) The power series function f(x) in 10(b) above may be written in terms of an old friend. Do so and reveal the *true identity* of f(x) in 10(b). (b) Obtain an infinite series that converges to the exact value of arctan(2).

(a) Although one obvious alias of f is

$$f(x) = \frac{\pi}{2} - \arctan(x) \ ,$$

a better moniker to mosey up to is

$$f(x) = \cot^{-1}(x) \ .$$

Even without the power series in hand, it is not difficult to see that f is the solution to the initial value problem

$$f'(x) = \frac{-1}{x^2+1} \ , \ \textit{with}\ \ f(0) = \frac{\pi}{2} \ .$$

Why is $\cot^{-1}(x)$ better?? Look at Part (b).

(b) What's the problem with arctan(2)? Just plug into the series for arctan, right?? Not so fast, Folks. The equation

$$\arctan(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1}$$

is only valid for real numbers in the closed interval [-1.1]. What are we to do?? It helps to remember that

$$\tan^{-1}(x) = \cot^{-1}(1/x)$$

for x > 0. So using the power series from Problem 10(b) now, we have

$$\arctan(2) = \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2} - \sum_{k=0}^{\infty} \frac{(-1)^k (1/2)^{2k+1}}{2k+1} \ .$$

But isn't $\tan^{-1}(x) = \pi/2 - \tan^{-1}(1/x)$ for x > 0, too??? Ok.