READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.*

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1. (10 pts.) Find Taylor's formula for the given function f at $a = \pi$. Find both the Taylor polynomial, $P_3(x)$, and the Lagrange form of the remainder term, $R_3(x)$, for the function $f(x) = \sin(x)$ at $a = \pi$. Then write $\cos(x)$ in terms of $P_3(x)$ and $R_3(x)$.

$P_3(x) =$

$R_3(x) =$

$\sin(x) =$

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2. (10 pts.) (a) Find the rational number represented by the following repeating decimal.

$0.2121212121\ldots =$

(b) Find all values of x for which the given geometric series converges, and then express the closed form sum of the series as a function of x.

$$\sum_{k=1}^{\infty} \frac{(x+1)^k}{5^k} =$$

3. (10 pts.) Using either comparison test or limit comparison test, determine whether each of the following series converges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2+5}{4n + n^5}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{24n-1}{n^2+n}$

4. (10 pts.) (a) Explain very briefly why integral test may not be used to show the series below is convergent.

$$\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$$

(b) Using only the integral test, determine whether the series below converges. Be explicit about the definition of the function f(x) used, and verify all the hypotheses of the theorem are true. [Warning: Details, details, details ...]

$$\sum_{n=1}^{\infty} \frac{2n}{n^4+1}$$

5. (10 pts.) Find the sums of each of the following convergent series. [Pay attention to the lower limits of summation, Folks.]

(a) $\sum_{n=2}^{\infty} \frac{1}{10^n} =$

(b) $\sum_{n=1}^{\infty} \frac{12}{n(n+2)}$

6. (5 pts.) Using divergence test, show that the series

$$\sum_{n=1}^{\infty} (-1)^n \left(\frac{n}{n+1}\right)$$

diverges. [Hint: A simple complex sentence with an embedded computation produces the desired magic.]

7. (5 pts.) It turns out the infinite series

$$\sum_{n=1}^{\infty} \frac{15}{n^4}$$

converges and has a sum that we shall denote by S. If you want a numerical estimate of S that is accurate to 2 decimal places, which partial sum,

$$S_N = \sum_{n=1}^{N} \frac{15}{n^4} ,$$

can you prove does the job? [Hint: There is an improper integral that provides an upper bound on the true error.]

8. (10 pts.) Find the radius of convergence and the interval of convergence of the power series function

$$\sum_{k=0}^{\infty} \frac{(-1)^k (x-3)^k}{(k+1)10^k} \ .$$

9. (10 pts.) (a) Apply the alternating series remainder estimate to estimate the error in approximating the sum of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$$

by the sum

$$\sum_{n=1}^{5} \frac{(-1)^{n+1}}{n^3} \ .$$

[Simply write an appropriate inequality.]

(b) Find a positive integer N such that the partial sum

$$\sum_{n=1}^{N} (-1)^{n+1}\left(\frac{5}{n^5}\right)$$

approximates the sum of the series

$$\sum_{n=1}^{\infty} (-1)^{n+1}\left(\frac{5}{n^5}\right)$$

to 4 decimal places, and prove your N actually does what you claim.

10. (15 pts.) (a) Using known power series, obtain a power series representation for the function $f(x) = (x - \arctan(x))/x^3$. Write your answer using sigma notation.

$$f(x) = \frac{x - \arctan(x)}{x^3} =$$

(b) Find a power series representation for the function f(x) below by doing termwise integration. Write your answer using sigma notation.

$$f(x) = \frac{\pi}{2} - \int_0^x \frac{1}{1 + t^2}\, dt =$$

(c) Beginning with the power series function

$$f(x) = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k x^k$$

defined for x satisfying $|x| < 2$, differentiate termwise to find the series representation for $f'(x)$. Write your answer using sigma notation.

$$f'(x) =$$

11. (5 pts.) With proof, determine whether the given series is conditionally convergent, absolutely convergent, or divergent.

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$$

10 Point Bonus: (a) The power series function f(x) in 10(b) above may be written in terms of an old friend. Do so and reveal the *true identity* of f(x) in 10(b). (b) Obtain an infinite series that converges to the exact value of arctan(2). [Say where your work is. It won't fit here.]