READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magical transformations, do not "box" your final results. Show me all the magic on the page, for I do not read minds.*

1. (5 pts.) State the Fundamental Theorem of Calculus.

2. (5 pts.) Complete the equation to write the following sum using sigma notation:

$$\frac{x^2}{1} - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} + \ldots - \frac{x^{44}}{22} =$$

3. (10 pts.) Evaluate each of the following sums in closed form.

(a) $\displaystyle\sum_{i=1}^{20} (i^2 + 1) =$

(b) $\displaystyle\sum_{i=0}^{99} \left(\frac{1}{5}\right)^i =$

4. (5 pts.) Using appropriate properties of the definite integral and suitable area formulas from geometry, evaluate the following definite integral. [Roughly sketching a couple of simple graphs might help. First use linearity, though.]

$$\int_{-2}^{2} x - (4 - x^2)^{1/2} \; dx =$$

5. (5 pts.)
Find the average value of $f(x) = \sec^2(x)$ over the interval $[-\pi/4,\pi/4]$.

$f_{AVE} =$

6. (5 pts.) Express the solution to the following initial value problem by using a definite integral with respect to the variable t,

$$\frac{dy}{dx} = \sec^3(x) \;,\quad y(1) = \pi.$$

To do this, fill in the right side of the equation below correctly. Do not attempt to evaluate the definite integral.

$y(x) =$

7. (5 pts.) Give an example of a bounded function defined on the closed interval [-1,1] that is not Riemann integrable.

8. (10 pts.) Assume that $[x_{i-1},x_i]$ denotes the *ith* subinterval of a partition of the interval [0,1] into n subintervals, all with the same length $\Delta x = 1/n$.

(a) Write the value of the following limit as a definite integral.

$$L = \lim_{n\to\infty} \sum_{i=1}^{n} \cos\left(\frac{\pi x_i}{4}\right)\Delta x$$

L =

(b) By evaluating the integral you obtained in part (a) above using the Fundamental Theorem of Calculus, give the exact numerical value of the limit, L.

L =

9. (10 pts.) Reveal all the details of evaluating the given integral by computing

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x,$$

where the sum is assumed to have originated from a regular partition of the given interval of integration. Here, you are to actually compute the Riemann sum in a closed form and then evaluate the limit. Reveal all the magical details.

$$\int_0^2 2x\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x =$$

10. (5 pts.) Using only the second comparison property of integrals, give both a lower and an upper bound on the true numerical value of the integral below.

$$I = \int_{\pi/3}^{\pi/2} \sin^2(x)\,dx$$

11. (5 pts.) A classmate showed you his solution to a certain integration problem where he made the substitution u = sin(x) so that du = cos(x)dx. What he did appears below.

$$\int_0^{\pi/2} \sin^2(x)\cos(x)\,dx = \int_0^{\pi/2} u^2\,du = \left(\frac{1}{3}u^3\right)\Big|_0^{\pi/2} = \left(\frac{1}{3}\sin^3(x)\right)\Big|_0^{\pi/2} = \frac{1}{3}$$

Briefly explain to him why he will not get full credit for his work if he submits it to his friendly Math Professor. Be specific.

12. (10 pts.) (a) Locate the critical points of the function g defined on the interval $[-\pi,\pi]$ by means of the equation

$$g(x) = \int_0^x t^2\cos(t)\,dt.$$

[Note: The critical points are actually in the open interval $(-\pi,\pi)$.]

(b) Determine the open intervals in $(-\pi, \pi)$ where g is increasing or decreasing.

13. (15 pts.) Differentiate the following functions:
[Make sure you label your derivatives correctly.]

(a) $g(x) = \int_{x}^{0} \frac{1}{(4+t^2)^{1/2}}\, dt$

(b) $f(x) = \int_{1}^{\sin(x)} \frac{1}{1+t^2}\, dt$

(c) $h(x) = \int_{x^3}^{\sin(x)} \cos(t)\, dt$

14. (5 pts.) Evaluate the following definite integral.

$\int_{0}^{2} | x^2 - 1 |\, dx =$

Silly 10 Point Bonus: Reveal all the magic in evaluating the following limit. [This is easy if you grok the proof of the hard part of the F.T.of C.]

$\lim_{x \to 0^+} \frac{1}{x} \int_{-x}^{x} \frac{\sec(t)}{\pi^2 + t^4}\, dt =$