READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page. Eschew obfuscation.*

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1. (10 pts.) Compute (a) the total distance, and (b) the net distance, traveled between time t = π/2 and time t = 3π/2 by a particle moving with the velocity function v = sin(t).

(a)

$$
\begin{aligned}
\text{Total\_Distance} &= \int_{\pi/2}^{3\pi/2} |\sin(t)|\,dt \\
&= \int_{\pi/2}^{\pi} |\sin(t)|\,dt + \int_{\pi}^{3\pi/2} |\sin(t)|\,dt \\
&= \int_{\pi/2}^{\pi} \sin(t)\,dt + \int_{\pi}^{3\pi/2} (-\sin(t))\,dt \\
&= (-\cos(t))\Big|_{\pi/2}^{\pi} + (\cos(t))\Big|_{\pi}^{3\pi/2} \\
&= -\cos(\pi) - (-\cos(\pi/2)) + \cos(3\pi/2) - \cos(\pi) = 2.
\end{aligned}
$$

(b)

$$
\begin{aligned}
\text{Net\_Distance} &= \int_{\pi/2}^{3\pi/2} \sin(t)\,dt \\
&= (-\cos(t))\Big|_{\pi/2}^{3\pi/2} \\
&= -\cos(3\pi/2) - (-\cos(\pi/2)) = 0.
\end{aligned}
$$

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2. (10 pts.) Differentiate each of the following functions.

(a) $f(x) = \log_{3\pi}(x)$ $\qquad f'(x) = \dfrac{1}{x\ln(3\pi)}$

(b) $f(x) = \sec^{-1}(x)$ $\qquad f'(x) = \dfrac{1}{|x|(x^2-1)^{1/2}}$

(c) $f(x) = 10^x$ $\qquad f'(x) = \ln(10)10^x$

(d) $f(x) = \sin^{-1}(x)$ $\qquad f'(x) = \dfrac{1}{(1-x^2)^{1/2}}$

(e) $f(x) = \cot^{-1}(x)$ $\qquad f'(x) = \dfrac{-1}{1+x^2}$

3. (10 pts.) Obtain the exact value of the arc length along the curve defined by the equation

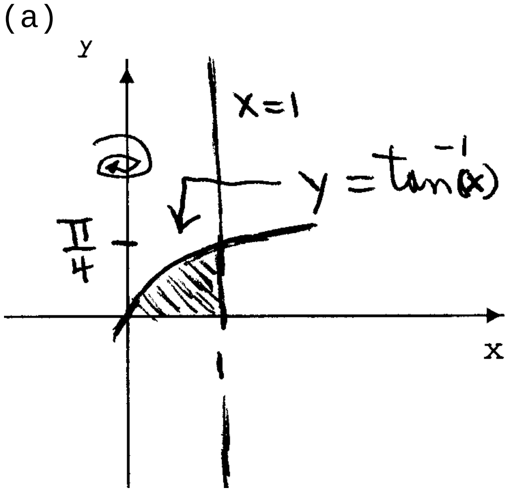
$$y = \frac{4x^{3/2}}{3}$$

from x = 2 to x = 6.

$$\begin{aligned}
L &= \int_2^6 \left(1 + \left(\frac{dy}{dx}\right)^2\right)^{1/2} dx \\
&= \int_2^6 (1+4x)^{1/2} dx \\
&= \frac{1}{4}\int_9^{25} u^{1/2} du \quad \textit{when} \quad u = 4x+1, \\
&= \frac{1}{4}\left(\frac{2u^{3/2}}{3}\right)\Big|_9^{25} \\
&= \frac{1}{6}[(25)^{3/2} - (9)^{3/2}] = \frac{49}{3}.
\end{aligned}$$

4. (10 pts.) (a) Sketch the region in the 1st quadrant enclosed by the curves defined by y = $\tan^{-1}(x)$, y = 0, and x = 1. Suppose the region is revolved around the y-axis. (b) Using the method of cylindrical shells, write down the definite integral used to compute the volume of the solid of revolution formed. **Don't evaluate the integral.** (c) Using the method of slicing [disks/washers here], write down the definite integral used to compute the same volume as in part (b). **Don't evaluate the integral.**

(a)



(b)

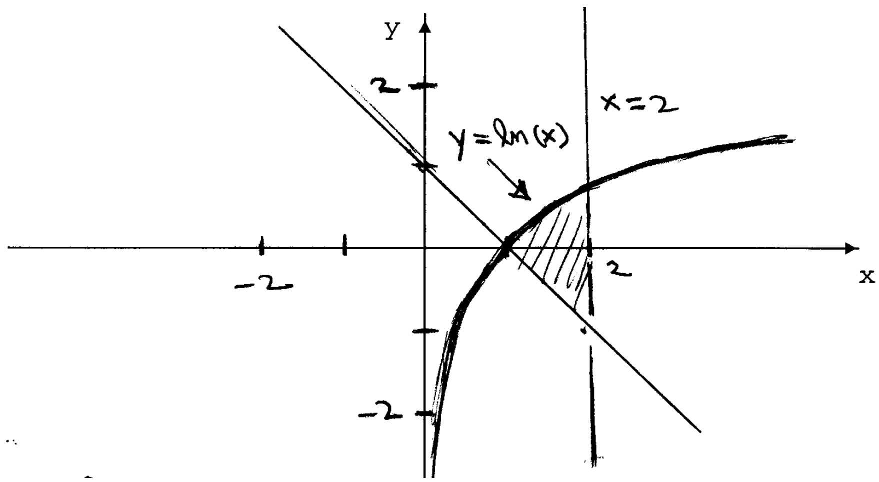
$$V = \int_0^1 2\pi x \tan^{-1}(x)\, dx$$

(c)

$$V = \int_0^{\pi/4} \pi(1)^2 - \pi \tan^2(y)\, dy$$

5. (10 pts.) (a) Sketch very carefully the bounded region bounded by the curves defined by y = ln(x), y = 1 - x, and x = 2 on the coordinate system provided. Label very carefully. (b) Simply write the definite integral, dx, that yields the area of the region. (c) Evaluate the definite integral of part (b) using the Fundamental Theorem of Calculus and any additional techniques of integration at your disposal.

(a)



(b) $\text{Area} = \int_1^2 \ln(x) - (1 - x)\, dx$

(c) $= \left(x\ln(x) - 2x + \frac{x^2}{2}\right)\Big|_1^2$

$= (2\ln(2) - 4 + 2) - \left(1\ln(1) - 2 + \frac{1}{2}\right)$

$= \ln(4) - \frac{1}{2}.$

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6. (10 pts.) (a) Using literal constants A, B, C, etc., write the form of the partial fraction decomposition for the proper fraction below. Do not attempt to obtain the actual numerical values of the constants A, B, C, etc. Do not attempt to integrate the rational function.

$$\frac{x^2+25}{(x-2)^3(x^2+1)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)^3} + \frac{Dx+E}{x^2+1} + \frac{Fx+G}{(x^2+1)^2}.$$

(b) If one were to integrate the rational function in part (a), at some point one would have to deal with the indefinite integral below. Reveal, in detail, how to evaluate this integral.

$$\int \frac{1}{(x^2+1)^2}\, dx = \int \frac{\sec^2(\theta)\, d\theta}{(\sec^2(\theta))^2}, \quad \textit{when } \tan(\theta) = x,$$

$$= \int \cos^2(\theta)\, d\theta = \int \frac{1+\cos(2\theta)}{2}\, d\theta = \frac{\theta}{2} + \frac{\sin(\theta)\cos(\theta)}{2} + C$$

$$= \frac{\tan^{-1}(x)}{2} + \frac{x}{2(x^2+1)} + C.$$

7. (30 pts.) Evaluate each of the following antiderivatives or definite integrals. Give exact values for definite integrals. [**5 pts./part**]

(a)

$$\int_0^{(\pi/6)^{1/2}} (8x)\cos(x^2)\,dx = \int_0^{\pi/6} 4\cos(u)\,du$$

$$= 4\sin(u)\Big|_0^{\pi/6} = 4\sin(\pi/6) - 4\sin(0) = 2$$

using $u = x^2$ *so that* $du = 2x\,dx$.

(b)

$$\int (8x)\cos(x)\,dx = 8x\sin(x) - \int 8\sin(x)\,dx$$

$$= 8x\sin(x) + 8\cos(x) + C$$

using parts: $u = 8x$ *and* $dv = \cos(x)dx$.

(c)

$$\int_0^{\pi/4} \frac{1}{\cos(x)}\,dx = \int_0^{\pi/4} \sec(x)\,dx$$

$$= \left(\ln|\sec(x) + \tan(x)|\right)\Big|_0^{\pi/4}$$

$$= \ln(\sqrt{2}+1) - \ln(1) = \ln(\sqrt{2}+1)$$

This was a hanging slider.

(d)

$$\int (1 - t^2)^{1/2}\,dt = \int (\cos^2(\theta))^{1/2}\cos(\theta)\,d\theta$$

$$= \int \cos^2(\theta)\,d\theta$$

$$= \int \frac{1+\cos(2\theta)}{2}\,d\theta$$

$$= \frac{\theta}{2} + \frac{\sin(2\theta)}{4} + C$$

$$= \frac{\sin^{-1}(t)}{2} + \frac{t(1-t^2)^{1/2}}{2} + C$$

by using the trig substitution $t = \sin(\theta)$ *so* $dt = \cos(\theta)d\theta$.

7. Evaluate each of the following antiderivatives or definite integrals. Give exact values for definite integrals.
[**5 pts./part**]
(e)

$$\int \sin(3x)\cos(5x)\,dx = \int \frac{\sin(8x)-\sin(2x)}{2}\,dx$$
$$= -\frac{1}{16}\cos(8x) + \frac{1}{4}\cos(2x) + C$$

by using $\sin(\alpha)\cos(\beta) = \frac{1}{2}(\sin(\alpha+\beta)+\sin(\alpha-\beta))$.

Using parts here is feasible but harder and messier.

(f)

$$\int_0^1 \ln(x^2+1)\,dx = \ln(2) - 2 + \frac{\pi}{2}$$

since

$$\int \ln(x^2+1)\,dx = x\ln(x^2+1) - \int \frac{2x^2}{x^2+1}\,dx$$
$$= x\ln(x^2+1) - 2\int 1 - \frac{1}{x^2+1}\,dx$$
$$= x\ln(x^2+1) - 2x + 2\tan^{-1}(x) + C$$

by using parts: $u = \ln(x^2+1)$ *and* $dv = 1dx$, *and then long division*.

8. (10 pts.) Consider the definite integral below. (a) Write down the sum, S_4, used to approximate the value of the integral below if Simpson's Rule is used with n = 4. **Do not attempt to evaluate the sum. Be very careful.** (b) Write down the sum, T_4, used to approximate the value of the integral below if Trapezoid Rule is used with n = 4. **Do not attempt to evaluate the sum. Be very careful.**

$$\int_3^4 x^{1/5}\,dx$$

Since $\Delta x = 1/4$, the endpoints of the regular partition we are using are $x_0 = 3$, $x_1 = 13/4$, $x_2 = 14/4$, $x_3 = 15/4$, and $x_4 = 4$.

(a) $S_4 = \frac{1}{12}\cdot(3^{1/5} + 4(13/4)^{1/5} + 2(14/4)^{1/5} + 4(15/4)^{1/5} + 4^{1/5})$

(b) $T_4 = \frac{1}{8}\cdot(3^{1/5} + 2(13/4)^{1/5} + 2(14/4)^{1/5} + 2(15/4)^{1/5} + 4^{1/5})$

Silly 10 Point Bonus: Let x > 0. Obtain the exact numerical value of the arc length of the garden variety parabola $y = t^2/2$ over the interval [0,x]. Work on the back of Page 4 of 5.
[Ogre Ogre's work found in c2-t2-bo.pdf.]