READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page. Eschew obfuscation.*

Silly 10 Point Bonus: Let $x > 0$. Obtain the exact numerical value of the arc length of the garden variety parabola $y = t^2/2$ over the interval $[0,x]$. Work on the back of Page 4 of 5.

This is actually a lob, a bit of Halloween trig or treat foolishness.

$$\begin{aligned} L &= \int_0^x \left(1 + \left(\frac{dy}{dt}\right)^2\right)^{1/2} dt \\ &= \int_0^x (1+t^2)^{1/2} dt \\ &= \int_0^{\tan^{-1}(x)} (1+\tan^2(\theta))^{1/2}\sec^2(\theta)\, d\theta \\ &= \int_0^{\tan^{-1}(x)} \sec^3(\theta)\, d\theta \end{aligned}$$

using the trig substitution $t = \tan(\theta)$ *so*

$dt = \sec^2(\theta)d\theta$ *and* $\theta = \tan^{-1}(t)$.

Then, using our bare neurons, in parts, we can obtain

$$\begin{aligned} \int \sec^3(\theta)\, d\theta &= \int \sec(\theta)\sec^2(\theta)\, d\theta \\ &= \sec(\theta)\tan(\theta) - \int (\sec(\theta)\tan(\theta))\tan(\theta)\, d\theta \\ &= \sec(\theta)\tan(\theta) - \int \sec(\theta)(\sec^2(\theta) - 1)\, d\theta \\ &= \sec(\theta)\tan(\theta) - \int \sec^3(\theta)\, d\theta + \int \sec(\theta)\, d\theta \\ &= \sec(\theta)\tan(\theta) - \int \sec^3(\theta)\, d\theta + \ln|\sec(\theta)+\tan(\theta)|. \end{aligned}$$

Here we clearly have put on our sun glasses to avoid uv exposure, set $u = \sec(\theta)$ and $dv = \sec^2(\theta)d\theta$, and then at an opportune time used the trig identity $1 + \tan^2(\theta) = \sec^2(\theta)$. By solving for the integral of $\sec^3(\theta)$ here, we have

$$\int \sec^3(\theta)\, d\theta = \frac{1}{2}(\sec(\theta)\tan(\theta) + \ln|\sec(\theta)+\tan(\theta)|) + C.$$

Since $\tan(\tan^{-1}(x)) = x$, and $0 < x$ implies $0 < \tan^{-1}(x) < \pi/2$, it follows that $\sec(\tan^{-1}(x)) = (1 + x^2)^{1/2}$. Thus, applying the Fundamental Theorem of Calculus, we see easily that

$$L = \frac{1}{2}[x(1+x^2)^{1/2} + \ln((1+x^2)^{1/2}+x)].$$

O.O.