
READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.*

Silly 10 Point Bonus: Obtain the closed form sum of the power series function in Problem 4 on Page 2 of 5. Say where your work is, for it won't fit here.

First, here is Problem 4 and its solution.

4. (10 pts.) Find the radius of convergence and the interval of convergence of the power series function

$$f(x) = \sum_{k=1}^{\infty} \frac{(-1)^k (x-2)^k}{k\,10^k} .$$

First observe that the series is centered at x = 2. Then we apply the ratio test for absolute convergence in order to determine the radius of convergence of the power series.

$$\rho(x) = \lim_{k\to\infty} \frac{|u_{k+1}|}{|u_k|} = \ldots = \lim_{k\to\infty} \frac{1}{10}\frac{k}{k+1}|x-2| = \frac{1}{10}|x-2|.$$

Now, $\rho(x) < 1$ if, and only if $|x - 2| < 10$. Thus, R = 10 is the radius of convergence. The endpoints are $x_L = -10 + 2$ and $x_R = 2 + 10$. When you substitute x_L into the power series and simplify the algebra, you obtain $\sum_{k=0}^{\infty} \frac{1}{k+1}$ which diverges. When you substitute x_R into the power series and simplify the algebra, you obtain $\sum_{k=0}^{\infty} \frac{(-1)^k}{k+1}$ which converges. I = (-8, 12].

Here is a "hard" solution to the bonus problem:

Observe that

$$f'(x) = \sum_{k=1}^{\infty} \frac{(-1)^k (x-2)^{k-1}}{10^k} = \sum_{j=0}^{\infty} \frac{-1}{10}\cdot\left(-\left(\frac{x-2}{10}\right)\right)^j = \frac{-1}{x+8}$$

provided $|x - 2| < 10$ or equivalently, -8 < x < 12. Clearly f(2) = 0. Thus, the function f satisfies the wee initial value problem, $f'(x) = -1/(x + 8)$ for $x \in (-8,12)$ with f(2) = 0. Using the Fundamental Theorem of Calculus, we have

$$f(x) = \int_2^x \frac{-1}{t+8}\,dt = \int_x^2 \frac{1}{t+8}\,dt = \ln(10) - \ln(x+8) = \ln\left(\frac{10}{x+8}\right)$$

for $x \in (-8,12)$.// (End of "hard" solution).

Here is an "easy" solution to the bonus problem:

Plainly,

$$\begin{aligned} f(x) &= \sum_{k=1}^{\infty} \frac{(-1)^k (x-2)^k}{k\,10^k} \\ &= \sum_{k=1}^{\infty} \frac{(-1)^k}{k}\left(\frac{x-2}{10}\right)^k \\ &= -\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k}\left(\frac{x-2}{10}\right)^k \\ &= -\ln\left(1 + \left(\frac{x-2}{10}\right)\right) \\ &= -\ln\left(\frac{x+8}{10}\right) = \ln\left(\frac{10}{x+8}\right) \end{aligned}$$

if $-1 < (x - 2)/10 \leq 1$ or $-8 < x \leq 12$. You need only have data for the Maclaurin series for $\ln(x + 1)$ stored in an accessible place in your bio-computer.//