READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.*

1. (10 pts.) Evaluate each of the following integrals. Look before you leap, for propriety may be problematical. Don't use the F.T. of C. if the integrand does not satisfy the required hypotheses.

(a)

$$\int_0^8 x^{-1/3}\,dx =$$

(b)

$$\int_{-\ln(\pi)}^{\infty} e^{-x}\,dx =$$

2. (10 pts.) Find Taylor's formula for the given function f at a = π. Find both the Taylor polynomial, $P_3(x)$, and the Lagrange form of the remainder term, $R_3(x)$, for the function $f(x) = \sin(x)$ at $a = \pi$. Then write $\sin(x)$ in terms of $P_3(x)$ and $R_3(x)$.

$P_3(x) =$

$R_3(x) =$

$\sin(x) =$

3. (10 pts.) Determine whether the sequence $\{a_n\}$ converges, and find its limit if it does.

(a) $a_n = \sum_{k=1}^{n} \left(\frac{3}{4}\right)^k$

(b) $a_n = 2\pi n \cdot \sin(\frac{4}{n})$

4. (10 pts.) Find the radius of convergence and the interval of convergence of the power series function

$$f(x) = \sum_{k=1}^{\infty} \frac{(-1)^k (x-2)^k}{k\,10^k} .$$

5. (10 pts.) Using only the integral test, determine whether the series below converges. Explicitly define the function f(x) used, and verify all the hypotheses of the theorem are true, including whether the function is decreasing.

$$\sum_{n=1}^{\infty} \frac{2}{n^2+n}$$

6. (10 pts.) (a) Find the rational number represented by the following repeating decimal.

0.212121212121 ... =

(b) Find all values of x for which the given geometric series converges, and then express the closed form sum of the series as a function of x.

$$\sum_{k=0}^{\infty} \frac{(-1)^k (x-2)^{k+1}}{10^k} =$$

7. (10 pts.) Using either comparison test or limit comparison test, determine whether each of the following series converges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\tan^{-1}(n)}{n^2+1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n+2)}{n^{1/2}}$

8. (5 pts.) Using only divergence test, show that the series

$$\sum_{n=1}^{\infty} n \tan\left(\frac{1}{2\pi n}\right)$$

diverges.

9. (5 pts) With proof, determine whether the given series is conditionally convergent, absolutely convergent, or divergent.

$\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{k^4}$

10. (15 pts.) (a) Using known power series, obtain a power series representation for the function $f(x) = \sin(x^2)$. Write your answer using sigma notation.

$f(x) = \sin(x^2) =$

(b) Beginning with the power series function

$$f(x) = \sum_{k=0}^{\infty} \left(\frac{1}{10}\right)^k x^k$$

defined for x satisfying $|x| < 10$, differentiate termwise to find the series representation for $f'(x)$. Write your answer using sigma notation.

$f'(x) =$

(c) Find a power series representation for the function f(x) below by doing termwise integration. Write your answer using sigma notation.

$f(x) = \int_0^x \frac{1}{1+t^2}\, dt =$

11. (5 pts.) Since the third and fourth Maclaurin polynomials for $\sin(x)$ are the same, it follows from Taylor's Theorem that if x is a real number different from 0, we may write

$$\sin(x) = x - \frac{1}{6}x^3 + \frac{\cos(c)}{120}x^5,$$

where c is some number between x and 0. Using this equation, obtain an open interval that is centered at 0 where $\sin(x)$ may be approximated to 1 decimal place accuracy using the polynomial

$$P_4(x) = x - (1/6)x^3.$$

Silly 10 Point Bonus: Obtain the closed form sum of the power series function in Problem 4 on Page 2 of 5. Say where your work is, for it won't fit here.