STUDENT NUMBER: **EXAM NUMBER:**

READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.*

1. (80 pts.) Obtain the exact numerical value of each of the following if possible. If a limit doesn't exist or an improper integral or an infinite series fails to converge, say how as precisely as possible. (10 pts./part)

(a) $\displaystyle\int_0^1 \frac{4}{(4-x^2)^{1/2}}\,dx =$

(b) $\displaystyle\int_0^{\pi/3} 4\theta\cos(\theta)\,d\theta =$

(c) $\displaystyle\int_{10}^{\infty} \frac{40}{x(x+1)}\,dx =$

1. (Continued) Obtain the exact numerical value of each of the following if possible... . (10 pts./part)

(d) $\lim_{n \to \infty} \sum_{k=10}^{n} \frac{40}{k(k+1)} =$

(e) $\int_{10}^{43} \frac{10}{(x-11)^{3/5}}\, dx =$

"Demon within!"

(f) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\sin(x)|\, dx =$

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1. (Continued) Obtain the exact numerical value of each of the following if possible... . (10 pts./part)

(g) $\lim_{n\to\infty} \ln\left(\frac{(n+1)(n+2)}{2n^2}\right) =$

(h) $\lim_{n\to+\infty} \frac{1}{n}\sum_{k=1}^{n} \frac{5k}{n} =$

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2. (10 pts.) Each of the following power series functions is the Maclaurin series of some well-known function. In each case, (i) identify the function, and (ii) provide the interval in which the series actually converges to the function.

(a) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!} =$

(b) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1} =$

(c) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!} =$

(d) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{k+1}}{k+1} =$

(e) $\sum_{k=0}^{\infty} \frac{x^k}{k!} =$

3. (30 pts.) Suppose that

$$f(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k\,4^k}(x-5)^k$$

(a) Find the radius of convergence and the interval of convergence of the power series function f.

(b) By using sigma notation and term-by-term differentiation *as done in class*, obtain a power series for $f'(x)$. What is the radius of convergence of the series for $f'(x)$?

$f'(x) =$

(c) Sum the series for $f'(x)$ to obtain $f'(x)$ in a closed form.

$f'(x) =$

(d) Obtain an infinite series whose sum is the same as the numerical value of the following definite integral. [We are working with the f of part (a). Use sigma notation and integrate term-by-term *as done in class*.]

$$\int_5^6 f(x)\,dx =$$

(e) Show how to compute $f^{(23)}(5)$ by using the fact that the Taylor series for f(x) at a = 5 is at the top of the page. [**Hint**: *You don't have to differentiate 23 times.*]

$f^{(23)}(5) =$

4. (30 pts.) (a) Using a complete sentence, state the first part of the Fundamental Theorem of Calculus.

(b) Using complete sentences, state the second part of the Fundamental Theorem of Calculus.

(c) Compute $g'(x)$ when $g(x)$ is defined by the following equation.

$$g(x) = \int_0^x e^{-t^2}\, dt + 7x^6$$

$g'(x) =$

(d) Write the following in terms of a definite integral with respect to the variable t: an antiderivative, $h(x)$, of the function $f(x) = e^{\tan(x)}$ with $h(\pi/4) = 0$. What is the natural domain of h(x) ??

$h(x) =$

(e) Write the solution to the following initial value problem in terms of a definite integral taken with respect to the variable **t**, so the differential denoting the variable of integration is **dt**. Do not attempt to evaluate the definite integral you get.

$$\frac{dy}{dx} = \tan^{-1}(x) \;\; ; \;\; y(1) = \sqrt{2}$$

$y(x) =$

5. (10 pts.) (a) Using literal constants A, B, C, etc., write the form of the partial fraction decomposition for the proper fraction below. Do not attempt to obtain the actual numerical values of the constants A, B, C, etc.

$$\frac{3x^2+5}{x^2(x-2)(4x^2+1)^2} =$$

(b) If one were to integrate the rational function in part (a), one probably would encounter the integral below. Reveal, in detail, how to evaluate this integral.

$$\int \frac{1}{4x^2+1}\,dx =$$

6. (10 pts.)
(a) Obtain the 3rd Taylor polynomial of $f(x) = \sin(\pi x)$ about $x_0 = 1/2$.

$p_3(x) =$

(b) By Using the Remainder Estimation Theorem, find an upper bound on the magnitude of error when $f(x) = e^x$ is approximated by the polynomial

$$p_n(x) = \sum_{k=0}^{n} \frac{x^k}{k!}$$

on the interval [-1,1]. Then use the upper bound on error that you get to determine how big n must be so that the polynomial will approximate $f(x)$ to 4 decimal place accuracy over the whole interval [-1,1]. [**Hint**: $2 < e < 3$.]

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7. (8 pts.) (a) Does alternating series test imply that

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}$$

converges conditionally?? Explain very briefly.

(b)
$$\frac{\pi}{4} = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}$$

For which $n \geq 1$, does the partial sum s_n approximate $\pi/4$ to 3 decimal places, where

$$s_n = \sum_{k=0}^{n} \frac{(-1)^k}{2k+1} \text{ ?}$$

Proof??

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8. (12 pts.) For each of the following multiple choice questions, circle the single most appropriate answer. Work on the back of page 6 if you must.

(i) The area of the region bounded by the graph of $f(x) = \sin(\pi x)$ and the x-axis between x = -1 and x = 1 is

(a) 0 (b) $2/\pi$ (c) $4/\pi$ (d) 4

(e) None of the above.

(ii) When the equation $(x - 12)^2 + (y + 5)^2 = 169$ in cartesian coordinates is transformed into an equation in the polar coordinates (r,θ), it may be placed in the form

(a) $r = 24\sin(\theta) - 10\cos(\theta)$ (b) $r = 10\cos(\theta) - 24\sin(\theta)$

(c) $r = 10\sin(\theta) - 24\cos(\theta)$ (d) $r = 24\cos(\theta) - 10\sin(\theta)$

(e) None of the above.

(iii) If $F(x) = \int_1^x x^2 t^{-1}\, dt$, then $F'(e) =$

(a) $2e$ (b) $3e$ (c) $-2e$ (d) e

(e) None of the above.

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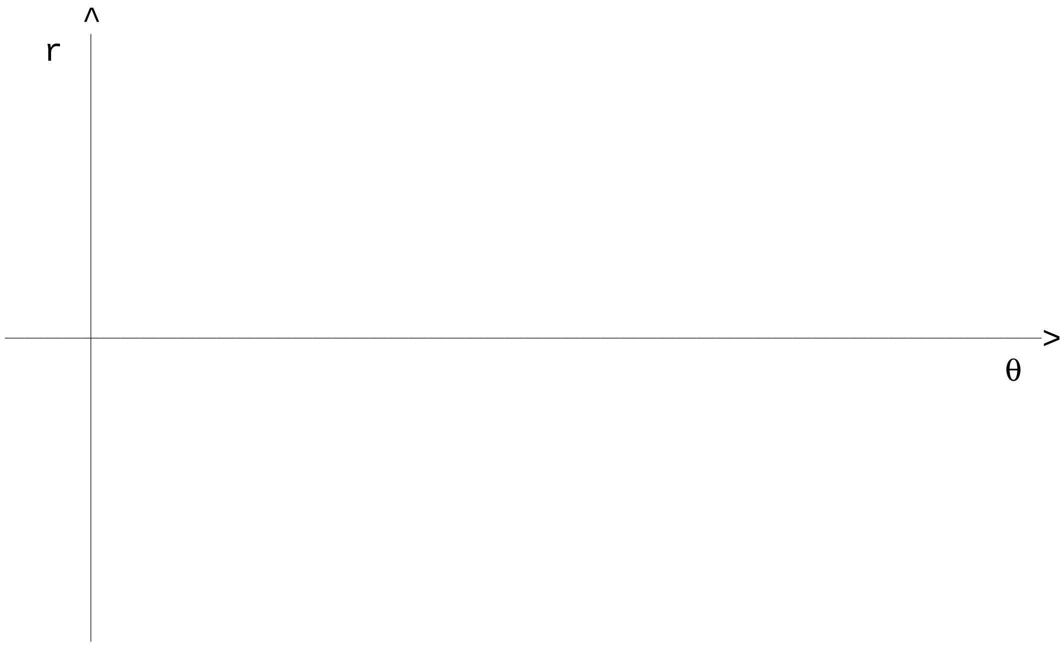
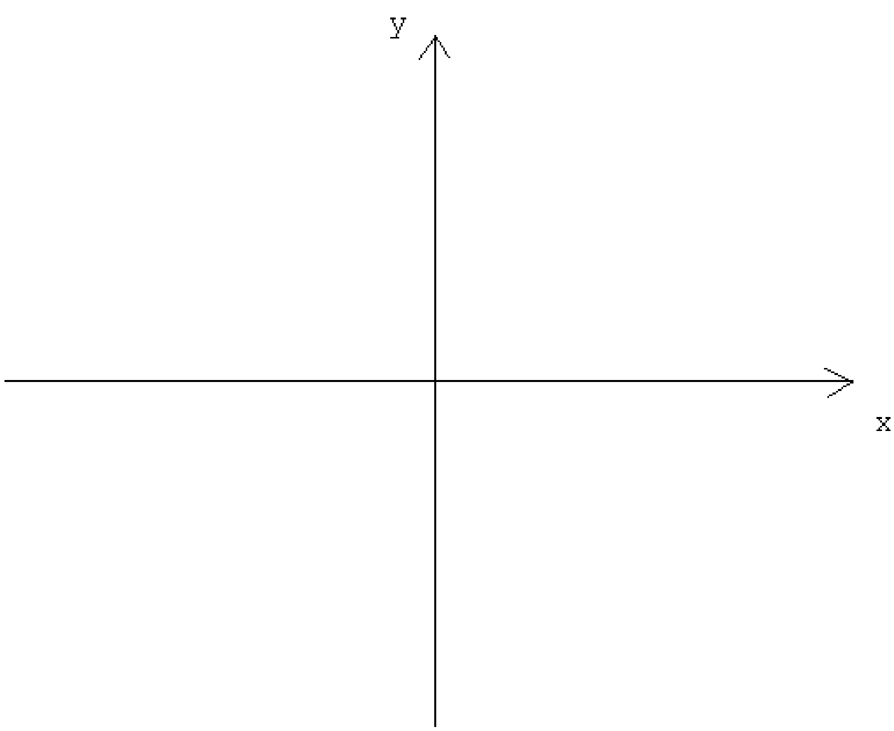
9. (10 pts.) Sketch the curve $r = 1 + 2\sin(\theta)$ in polar coordinates. Do this as follows: (a) Carefully sketch the auxiliary curve, a rectangular graph, on the r,θ-coordinate system provided. (b) Then translate this graph to the polar one.

(a)



(b) [Think of polar coordinates overlaying the x,y axes below.]



Silly 10 point bonus: Obtain the area enclosed by the inner loop of the limaçon above -- the one defined by the equation $r = 1 + 2\sin(\theta)$. Say where your work is, for it won't fit here.