READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page, for I do not read minds. Eschew obfuscation.*

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1. (5 pts.) Express the following sum using sigma notation, but do not attempt to find its numerical value.

$$\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \frac{1}{7} =$$

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2. (5 pts.) Obtain the upper limit of summation and rewrite the summand in order to make the following equation true. **Do not attempt to evaluate the sum.**

$$\sum_{k=4}^{45} k^2 = \sum_{j=1}$$

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3. (5 pts.) Express the following limit as a definite integral. Do not attempt to evaluate the integral.

$$L = \lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} 4x_k^*(1 - 3(x_k^*)^2)\Delta x_k \ ; \quad a = -4,\ b = 1.$$

L =

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4. (10 pts.) If the function f is continuous on [a,b], then the *net signed area* A between $y = f(x)$ and the interval [a,b] is defined by

$$A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k^*)\Delta x.$$

Reveal all the details in obtaining the numerical value of the net signed area of $f(x) = x^2$ over the interval [0,1] using the definition above with

$$x_k^*$$

the right end point of each subinterval in the regular partition.

5. (5 pts.) Using a complete sentence and appropriate notation, state precisely the First Part of the Fundamental Theorem of Calculus. [This is sometimes called the *Evaluation Theorem*.]

6. (5 pts.) Evaluate the following using the 1st Part of the Fundamental Theorem of Calculus.

$$\int_{-1}^{1} \frac{1}{1+x^2}\, dx =$$

7. (5 pts.) Using complete sentences and appropriate notation, state the Second Part of the Fundamental Theorem of Calculus.

8. (10 pts.) A particle moves with a velocity of $v(t) = t^2 - 1$ along an s-axis. Find the displacement and total distance traveled over the time interval [0,2].

Displacement =

Total_Distance =

9. (5 pts.) Write the solution to the following initial value problem in terms of a definite integral taken with respect to the variable **t**, so the differential denoting the variable of integration is **dt**.

$$\frac{dy}{dx} = \sec^2(x)e^{\tan(x)} \quad \text{with } y(\pi/4) = 5$$

$y(x) =$

10. (5 pts.) If the given integral is expressed as an equivalent integral in terms of the variable u using the substitution $u = x^2 - 1$ so that the equation below is true, what are the numerical values of the new limits of integration α and β, and what is the new integrand, f(u) ??? Obtain these but do not attempt to evaluate the du integral.

$$\int_0^2 |x^2-1|2x\,dx = \int_\alpha^\beta f(u)\,du$$

$\alpha =$ $\beta =$ $f(u) =$

11. (5 pts.) (a) (3 pts.) Give the definition of the function ln(x) in terms of a definite integral and give its domain and range. Label correctly. (**Hint**: Complete the sentence,"ln(x) = ...")

(b) (2 pts.) Given ln(a) = -20 and ln(c) = 5, evaluate the following integral.

$$\int_{c^5}^{a} \frac{1}{\lambda}\,d\lambda =$$

12. (10 pts.) Find each of the following derivatives.

(a) $\frac{d}{dx}\left[\int_1^x \frac{\sin(t)}{t}\,dt\right] =$

(b) $\frac{d}{dx}\left[\int_1^{\tan(x)} \tan^{-1}(t)\,dt + x^2\right] =$

13. (5 pts.) Evaluate the following limit:

$$L = \lim_{x \to \infty} \left(1 + \frac{1}{3x}\right)^{2x} =$$

14. (10 pts.) Evaluate the following two sums in closed form.

(*a*) $\displaystyle\sum_{k=2}^{20} \left(\frac{1}{k^2} - \frac{1}{(k-1)^2} \right) =$

(*b*) $\displaystyle\sum_{k=1}^{20} 3^k =$

15. (10 pts.) Let the function g be defined by the equation

$$g(x) = \int_2^x (3t^2+1)^{1/2}\, dt$$

for $x \in (-\infty, \infty)$. Then

(a) $g(2) =$

(b) $g'(2) =$

(c) $g''(2) =$

(d) Determine the open intervals where g is increasing or decreasing. Be specific.

(d) Determine the open intervals where g is concave up or concave down. Be specific.

Silly 10 Point Bonus: Show how to use the Mean-Value Theorem of Differential Calculus to prove that

$$\frac{1}{x+1} < \int_x^{x+1} \frac{1}{t}\, dt < \frac{1}{x}$$

for each $x > 0$. [Say where your work is, for it won't fit her.]