READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page. Eschew obfuscation.*

1. (10 pts.) Obtain the second Taylor polynomial $p_2(x)$ of the function

$$f(x) = x^{1/2}$$

at $x_0 = 4$.

2. (10 pts.) Suppose that

$$f(x) = \sum_{k=1}^{\infty} \frac{(-1)^k}{k\,10^k}(x - 1)^k$$

Find the radius of convergence and the interval of convergence of the power series function f.

3. (10 pts.) Each of the following power series functions is the Maclaurin series of some well-known function. In each case, (i) identify the function, and (ii) provide the interval in which the series actually converges to the function.

(a) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!} =$

(b) $\sum_{k=0}^{\infty} \frac{x^k}{k!} =$

(c) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1} =$

(d) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!} =$

(e) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{k+1}}{k+1} =$

4. (5 pts.) Express .272727... (repeating) as the ratio of two positive integers. [**The ratio does not have to be in lowest terms.**]

.272727... =

5. (5 pts.) Prove the infinite series below is conditionally convergent. [Helpful Hint ?? : $\ln(k) < k$ when $k \geq 1$.]

$$\sum_{k=2}^{\infty} \frac{(-1)^{k+1}}{\ln(k)}$$

6. (5 pts.) Using root test, determine whether the following series converges.

$$\sum_{k=1}^{\infty} \left(\frac{2}{1 + \frac{1}{k^2}} \right)^k$$

7. (5 pts.) Using comparison test or limit comparison test, determine whether the following series converges.

$$\sum_{k=1}^{\infty} \frac{10k+1}{2+3k^3}$$

8. (5 pts.) Does divergence test tell you anything about the following series? Explain briefly. Note: You do not actually have to determine whether the series converges.

$$\sum_{k=1}^{\infty} \frac{10\ln(k)}{2+3k}$$

9. (5 pts.) Using integral test, determine whether the following series converges. [Hint??: Begin by defining f(x) appropriately.]

$$\sum_{k=2}^{\infty} \frac{4}{k\ln(k)}$$

10. (20 pts.) Obtain the exact numerical value of each of the following if possible. If a limit doesn't exist or an improper integral or an infinite series fails to converge, say so.

(a) $\int_1^{\infty} e^{-x}\,dx =$

(b) $\sum_{k=1}^{\infty} e^{-k} =$

(c) $\int_1^{2} \frac{1}{(2-x)^{1/2}}\,dx =$

(d) $\lim_{n\to\infty} \cos\left(\frac{\ln(n)}{n}\right) =$

11. (6 pts.) Suppose

$$f(x) = \sum_{k=1}^{\infty} \frac{\pi(x-5)^k}{k\,20^k}$$

for every $x \in (-15,25)$. By differentiating f term-by-term, obtain a power series function that is the same as f′(x). **Use sigma notation as in class.**

$f'(x) =$

12. (6 pts.) **Using sigma notation as in class and an appropriate Maclaurin series**, by doing term-by-term integration, obtain an infinite series that is equal to the numerical value of the following definite integral.

$$\int_0^1 e^{-x^2}\,dx =$$

13. (8 pts.) (a) By substitution into an appropriate Maclaurin series, obtain the Maclaurin series for the function

$$f(x) = \frac{18}{9+x^2}$$

[**Hint**: *You must do a little algebra before substituting. Expect to meet an old friend??*]

(b) What is the domain of the function f?

(c) What is the interval of convergence for the Maclaurin series of f??

Silly 10 Point Bonus: Show how to find an interval that is symmetric about the origin where $\sin(x)$ can be approximated by $p(x) = x - x^3/6$ with two decimal place accuracy. [Indicate where your work is, for it won't fit here.]