STUDENT NUMBER: EXAM NUMBER:

READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.*

1. (30 pts.) (a) Using a complete sentence, state the first part of the Fundamental Theorem of Calculus.

(b) Using complete sentences, state the second part of the Fundamental Theorem of Calculus.

(c) Compute $g'(x)$ when $g(x)$ is defined by the following equation.

$$g(x) = \int_0^x \frac{1}{\sqrt{1 - t^2}}\, dt + \sin^2(x)$$

$g'(x) =$

(d) Give the definition of the function $\ln(x)$ in terms of a definite integral and give its domain and range. Label correctly.

(e) Write the solution to the following initial value problem in terms of a definite integral taken with respect to the variable t, so the differential denoting the variable of integration is dt. Then reveal an alternative identity of y by evaluating the definite integral with respect to t.

$$\frac{dy}{dx} = \frac{\ln(x)}{x} \quad \text{with} \quad y(e) = 5.$$

$y(x) =$

2. (30 pts.) Here are five easy antiderivatives to evaluate.

(a) $\int \frac{\cos^2(x)}{\sin(x)}\, dx =$

(b) $\int 2x \ln(x)\, dx =$

(c) $\int 2 \sin^2(x)\, dx =$

(d) $\int (2x+1)e^x\, dx =$

(e) $\int \frac{4x^4+4x^2+x+36}{x^2+1}\, dx =$

3. (18 pts.) Suppose

$$f(x) = \sum_{k=1}^{\infty} \frac{5(-1)^k}{\sqrt{k}\, 8^k}(x-2)^k$$

(a) Find the radius of convergence and the interval of convergence of the power series function *f*.

(b) By using sigma notation and term-by-term differentiation *as done in class*, obtain a power series for $f'(x)$. What is the radius of convergence of the series for $f'(x)$?

$f'(x) =$

(c) By using sigma notation and integrating as done in class, obtain an infinite series whose sum is the same as the numerical value of the following definite integral. [We are working with the power series *f* of part (a).]

$$\int_2^4 f(x)\, dx =$$

4. (12 pts.) Each of the following power series functions is the Maclaurin series of some well-known function. In each case, (i) identify the function, and (ii) provide the interval in which the series actually converges to the function.

(a) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k x^{k+1}}{k+1} =$

(b) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1} =$

(c) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!} =$

(d) $\displaystyle\sum_{k=0}^{\infty} \frac{x^k}{k!} =$

(e) $\displaystyle\sum_{k=0}^{\infty} x^k =$

(f) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!} =$

5. (18 pts.) (a) Using literal constants A, B, C, etc., write the form of the partial fraction decomposition for the proper fraction below. Do not attempt to obtain the actual numerical values of the constants A, B, C, etc.

$$\frac{3x^2+5}{x(x-2)(x^2+1)^3} =$$

(b) If one were to integrate the rational function in part (a), one probably would encounter the integral below. Find this integral.

$$\int \frac{x}{(x^2+1)^3}\,dx =$$

(c) Obtain the numerical values of the literal constants *A*, *B*, and *C* in the partial fraction decomposition below.

$$\frac{x-2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

6. (12 pts.) (a) Find rectangular coordinates of the point whose polar coordinates are given below.

$$(r,\theta) = (-7, 5\pi/3)$$

(b) The following point is given in rectangular coordinates. Express the point in polar coordinates with $r \geq 0$ and $0 \leq \theta < 2\pi$.

$$(x,y) = (-2\sqrt{3}, 2)$$

(c) Identify the given curve after transforming its polar equation into one in rectangular coordinates.

$$4r\cos(\theta) - 2r\sin(\theta) = 12$$

(d) Transform the given rectangular equation into an equivalent one in polar coordinates and identify the curve completely.

$$x^2 + y^2 - 10y = 0$$

7. (15 pts.) Classify each of the following series as absolutely convergent, conditionally convergent, or divergent. Note that no proofs are required!! Consequently, be very careful.

(a) $\sum_{k=1}^{\infty} (-1)^k \left(\frac{k^{3/8}}{k^{1/4}+1} \right)$

(b) $\sum_{k=1}^{\infty} (-1)^k \left(\frac{k}{k^2+1} \right)$

(c) $\sum_{k=1}^{\infty} \frac{(-1)^k}{2^k}$

8. (15 pts.) Here are three convergent infinite series that should be easy to sum up at this stage. Obtain their sums.

(a) $\sum_{k=1}^{\infty} \frac{10}{k(k+1)} =$

(b) $\sum_{k=0}^{\infty} 10 \left(\frac{1}{2} \right)^k =$

(c) $\sum_{k=0}^{\infty} \frac{10(\ln(3))^k}{k!} =$

9. (10 pts.) Obtain the 3rd Taylor polynomial of $f(x) = \cos(x)$ about $x_0 = \pi/2$.

$p_3(x) =$

10. (10 pts.) Find the area under the curve

$$y = \sqrt{4 - x^2}$$

from x = -1 to x = 1. Hint: ’Tis not a semi-circle, folks.

$Area =$

11. (10 pts.) (a) Evaluate the following improper integral.

$$\int_1^{\infty} \frac{x}{x^2+1}\,dx =$$

(b) Does part (a) of this problem help tell you anything about the following series? Explain briefly.

$$\sum_{k=1}^{\infty} \frac{k}{k^2+1}$$

12. (10 pts.) (a) The sum of the alternating harmonic series is ln(2). Use the error estimate from alternating series test to determine a specific value of $n \geq 1$ so that the partial sum s_n approximates ln(2) to 6 decimal places, where, of course,

$$s_n = \sum_{k=1}^{n} \frac{(-1)^{k-1}}{k}.$$

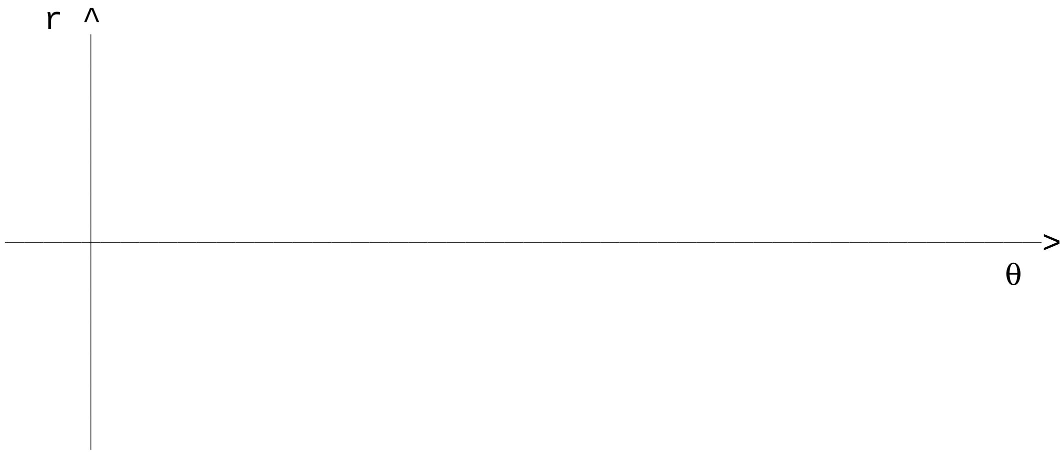
(b) Use the Remainder Estimation Theorem to obtain an upper bound on the error of the approximation

$$\cos(x) \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

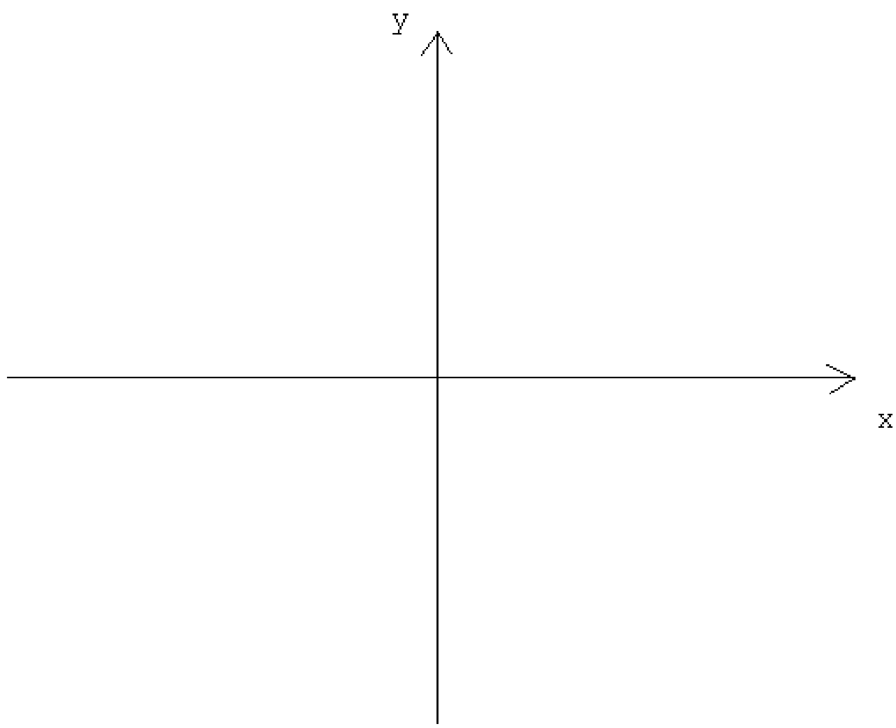
when $0 \leq x \leq 1$.

13. (10 pts.) Sketch the curve $r = 4\cos(2\theta)$ in polar coordinates.
Do this as follows: (a) Carefully sketch the auxiliary curve, a rectangular graph, on the r,θ-coordinate system provided. (b) Then translate this graph to the polar one.

(a)



(b) [Think of polar coordinates overlaying the x,y axes below.]



(c) Write down, but do not attempt to evaluate a definite integral that provides the numerical value of the area of one of the rose petals above.

Area =

Silly 10 point bonus: State the Mean-Value Theorem for Integrals and use it to prove the second part of the Fundamental Theorem of Calculus. Say where your work is, for it won't fit here.