READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page, for I do not read minds. Eschew obfuscation.*

1. (5 pts.) Express the following sum using sigma notation, but do not attempt to find its numerical value.

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} =$$

2. (5 pts.) Obtain the upper limit of summation and rewrite the summand in order to make the following equation true. **Do not attempt to evaluate the sum.**

$$\sum_{k=1}^{45} k^4 = \sum_{j=10}$$

3. (5 pts.) Using a complete sentence and appropriate notation, state precisely the First Part of the Fundamental Theorem of Calculus. [This is sometimes called the *Evaluation Theorem*.]

4. (5 pts.) Evaluate the following using the 1st Part of the Fundamental Theorem of Calculus.

$$\int_{-\pi/4}^{\pi/4} \cos(x)\, dx =$$

5. (5 pts.) Express the following limit as a definite integral. Do not attempt to evaluate the integral.

$$L = \lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} (\sin^2 x_k^*)\Delta x_k \ ; \quad a = 0,\ b = \pi/2.$$

L =

6. (5 pts.) Using complete sentences and appropriate notation, state the Second Part of the Fundamental Theorem of Calculus.

7. (10 pts.) Find each of the following derivatives.

(a) $\frac{d}{dx}\left[\int_1^x \sin(t^2)\, dt\right] =$

(b) $\frac{d}{dx}\left[\int_0^{e^{-x}} \frac{1}{1+t^4}\, dt\right] =$

8. (5 pts.) (a) (3 pts.) Give the definition of the function ln(x) in terms of a definite integral and give its domain and range. Label correctly. (**Hint**: Complete the sentence,"ln(x) = ...")

(b) (2 pts.) Given ln(a) = 2 and ln(b) = -3, evaluate the following integral.

$\int_1^{ab^2} \frac{1}{t}\, dt =$

9. (5 pts.) Write the solution to the following initial value problem in terms of a definite integral taken with respect to the variable **t**, so the differential denoting the variable of integration is **dt**. DO NOT ATTEMPT TO EVALUATE THE DEFINITE INTEGRAL IN *t* THAT YOU OBTAIN.

$$\frac{dy}{dx} = \frac{2x^2+1}{x}, \quad y(1) = 2.$$

$y(x) =$

10. (5 pts.) Evaluate the definite integral below by expressing it in terms of u correctly and then evaluating the resulting integral using a formula from geometry.

$$\int_{-5/3}^{5/3} \sqrt{25-9x^2}\; dx \;;\; u = 3x$$

$\int_{-5/3}^{5/3} \sqrt{25-9x^2}\; dx =$

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11. (10 pts.) Write each of the following two sums in closed form.

(*a*) $\sum_{k=1}^{n} \frac{3k}{n} =$

(*b*) $\sum_{k=0}^{n} \frac{1}{2^k} =$

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12. (10 pts.) If the function *f* is continuous on [a,b], then the *net signed area* A between $y = f(x)$ and the interval [a,b] is defined by

$$A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k^*)\Delta x.$$

Reveal all the details in computing the numerical value of the net signed area of $f(x) = x^2$ over the interval [0,2] using only the definition above with

$$x_k^*$$

the right end point of each subinterval in the regular partition.

13. (10 pts.) A particle moves with a velocity of $v(t) = \cos(t)$ along an s-axis. Find the displacement and total distance traveled over the time interval $[\pi/2, 2\pi]$.

Displacement =

Total_Distance =

14. (5 pts.) Evaluate the following limit:

$$L = \lim_{x \to \infty} \left(1 + \frac{1}{2x}\right)^{x} =$$

15. (10 pts.) Let the function g be defined by the equation

$$g(x) = \int_0^x \cos^4(t)\, dt + 2$$

for $x \in (-\pi/2 , \pi/2)$. Then

(a) $g(0) =$

(b) $g'(0) =$

(c) $g''(0) =$

(d) Determine the open intervals where g is increasing or decreasing. Be specific.

(e) Determine the open intervals where g is concave up or concave down. Be specific.

Silly 10 Point Bonus: Obtain the exact numerical value of the limit L in Problem 5 on Page 1 of 4. [Say where your work is, for it won't fit here.]