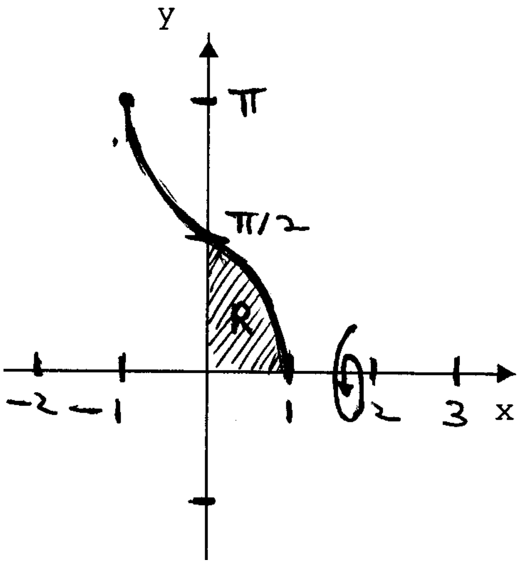
READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.*

1. (25 pts.) The region in the first quadrant enclosed by the curves $y = \cos^{-1}(x)$, $x = 0$, and $y = 0$ is sketched below for your convenience.



(a) Write down, but do not attempt to evaluate the definite integral whose numerical value gives the area of the region R if one integrates with respect to x so the differential in the integral is dx.

Area =

(b) Write down, but do not attempt to evaluate the definite integral whose numerical value gives the area of the region R if one integrates with respect to y so the differential in the integral is dy.

Area =

(c) Using the method of disks or washers, write a single definite integral dx whose numerical value is the volume of the solid obtained when the region R above is revolved around the x-axis. Do not evaluate the integral.

Volume =

(d) Using the method of cylindrical shells, write down a definite integral dy to compute the same volume as in part (c). Do not evaluate the integral.

Volume =

(e) Write down, but do not attempt to evaluate, the definite integral that gives the arc-length of the curve $y = \cos^{-1}(x)$ from $x = 0$ to $x = 1/2$.

Length =

2. (15 pts.) (a) (10 pts.) Using literal constants A, B, C, etc., write the form of the partial fraction decomposition for the proper fraction below. Do not attempt to obtain the actual numerical values of the constants A, B, C, etc. Be very careful here.

$$\frac{4x^2+5}{(x+1)^3(9x^2+4)^2} =$$

(b) (5 pts.) If one were to integrate the rational function in part (a), one probably would encounter the integral below. Reveal, in detail, how to evaluate this integral.

$$\int \frac{1}{9x^2+4}\,dx =$$

3. (60 pts.) Evaluate each of the following antiderivatives or definite integrals. Give exact values for definite integrals.
[**6 pts./part**]

(a) $\int 2x\cos(x)\,dx =$

(b) $\int_0^{(\pi/2)^{1/2}} 2x\sin(x^2)\,dx =$

3. (Continued) Evaluate each of the following antiderivatives or definite integrals. Give exact values for definite integrals.
[**6 pts./part**]

(c) $\int \tan^{-1}(x)\, dx =$

(d) $\int x^2 e^x dx =$

(e) $\int \frac{\sin^2(t)}{\cos(t)}\, dt =$

(f) $\int \sqrt{1 - x^2}\, dx =$

3. (Continued) Evaluate each of the following antiderivatives or definite integrals. Give exact values for definite integrals.

(g) $\int_0^1 \frac{2x^4+4x^3}{x^2+1}\,dx =$

(h) $\int \sin(4t)\cos(2t)\,dt =$

(i) $\int \frac{1}{x^2-1}\,dx =$

(j) $\int \cos(x)e^x\,dx =$

Silly 10 Point Bonus: What magical theorem ensures that all real functions f that are continuous on an interval, I, have real honest-to-goodness antiderivatives that are alive and well on the interval, I?? State and prove the magical theorem.//Say where your work is, for there isn't room here.