READ ME FIRST: Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page. Eschew obfuscation.

1. (6 pts.) (a) Using complete sentences and appropriate notation, give the precise ε - N definition of

$$\lim_{n \to \infty} a_n = L.$$

(b) Give the precise mathematical definition of the sum of an infinite series,

$$\sum_{k=1}^{\infty} a_k$$

2. (2 pts.) Suppose that

$$\sum_{n=1}^{\infty} a_n = -\frac{2}{15} \qquad \text{and} \qquad \sum_{n=1}^{\infty} b_n = 6.$$

Then we have

$$\sum_{n=1}^{\infty} \left(60a_n + \frac{5}{3}b_n \right) =$$

3. (12 pts.) (a) [Complete the following.] A p-series is a series of the form

$$\sum_{k=1}^{\infty}$$

This series converges if _______________ and this series diverges if _______________.

(b) [Complete the following.] A geometric series is a series of the form

$$\sum_{k=0}^{\infty}$$

This series converges if _______________ and this series diverges if _______________.

4. (5 pts.) Consider the definite integral below. Write down the sum, S_4, used to approximate the value of the integral below if Simpson's Rule is used with $n = 4$. **Do not attempt to evaluate the sum. Be very careful.**

$$\int_1^3 x^{1/4}\, dx$$

S_4 =

5. (5 pts.) Make an appropriate u-substitution of the form $u = x^{1/n}$ or $u = (x + a)^{1/n}$ and then evaluate the integral.

$$\int \frac{dx}{x(1 - x^{1/4})} =$$

6. (5 pts.) Evaluate the following integral using the substitution $u = \tan(x/2)$.

$$\int \frac{1}{1 - \sin(x)}\, dx =$$

7. (10 pts.) Evaluate the integrals that converge.

(a) $$\int_1^{+\infty} \frac{1}{x\sqrt{x^2 - 1}}\, dx =$$

(b) $$\int_0^1 \frac{1}{(1-x)^{2/3}}\, dx =$$

8. (4 pts.) Find the general term of the sequence, starting with n = 1, determine whether the sequence converges, and if so, find its limit.

$$(\sqrt{2} - \sqrt{3}),\ (\sqrt{3} - \sqrt{4}),\ (\sqrt{4} - \sqrt{5}),\ \ldots.$$

9. (4 pts.) Express the repeating decimal as a fraction.

$0.141414\ldots =$

10. (4 pts.) [Complete the following.] The harmonic series has the form

$$\sum_{k=1}^{\infty}$$

What is the sum of the harmonic series?

11. (8 pts.) Determine whether the series converges, and if so, find its sum.

(*a*) $$\sum_{k=1}^{\infty}\left(-\frac{4}{3}\right)^{k-1}$$

(*b*) $$\sum_{k=1}^{\infty}\frac{2}{(k+3)(k+4)}$$

12. (5 pts.) Find all values of x for which the series converges, and find the sum of the series for those values of x.

$$e^{-x} + e^{-2x} + e^{-3x} + e^{-4x} + e^{-5x} + \ldots$$

13. (4 pts.) Use ratio test to determine whether the series converges, If the test is inconclusive, say so.

$$\sum_{k=1}^{\infty} \frac{k!}{k^3}$$

14. (4 pts.) Use root test to determine whether the series converges. If the test is inconclusive, say so.

$$\sum_{k=1}^{\infty} \frac{k}{5^k}$$

15. (4 pts.) Use comparison test to show the following series converges.

$$\sum_{k=1}^{\infty} \frac{5\sin^2(k)}{k^2}$$

16. (4 pts.) Apply the divergence test and state what it tells you about each of the following series.

(a) $\sum_{k=1}^{\infty} \frac{1}{k!}$

(b) $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{\sqrt{k}+3}$

17. (4 pts.) Confirm that the integral test is applicable and then use it to determine whether the following series converges:

$$\sum_{k=1}^{\infty} \frac{k}{1+k^2}$$

18. (6 pts.) Consider the sequence

$$a_1 = \sqrt{6}$$

$$a_2 = \sqrt{6 + \sqrt{6}}$$

$$a_3 = \sqrt{6 + \sqrt{6 + \sqrt{6}}}$$

$$a_4 = \sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6}}}}$$

$$\vdots$$

(a) Find a recursion formula for a_{n+1}.

(b) Assuming the sequence converges, find the limit, L.

19. (4 pts.) From the definition of a limit of sequence, we know there is a positive integer N so that if $n \geq N$, then

$$\left| \frac{5n}{n+3} - 5 \right| < .0008$$

since

$$\lim_{n \to \infty} \frac{5n}{n+3} = 5.$$

Find a positive integer N which works and prove it provides the desired error bound. [You don’t have to get the best. Just obtain one the you can prove works, and then convince the skeptics.]

Silly 10 Point Bonus: Prove that the sequence $\{a_n\}$ of Problem 18 actually converges. Say where your work is, for it won’t fit here.