NAME:

STUDENT NUMBER: EXAM NUMBER:

Read Me First:

Read each problem carefully and do exactly what is requested. Full credit will be awarded only if you show all your work neatly, and it is correct. Use complete sentences and use notation correctly. Remember that what is illegible or incomprehensible is worthless. Communicate. Eschew obfuscation. Good Luck! [Total Points Possible: 120]

1. (20 pts.) Here are five easy antiderivatives to evaluate.

(a) $\int 9x^2 \ln(x)\ dx =$

(b) $\int \frac{1}{x^2+x}\ dx =$

(c) $\int \frac{\sin^2(x)}{\cos(x)}\ dx =$

(d) $\int \frac{3x^4+3x^2+2x+2}{x^2+1}\ dx =$

(e) $\int (x^2+x)\, e^x\ dx =$

2. (10 pts.) Each of the following power series functions is the Maclaurin series of some well-known function. In each case, (i) identify the function, and (ii) provide the interval in which the series actually converges to the function.

(a) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!} =$

(b) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1} =$

(c) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!} =$

(d) $\sum_{k=0}^{\infty} \frac{x^k}{k!} =$

(e) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{k+1}}{k+1} =$

3. (5 pts.) Suppose that

$$f(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k\,20^k}(x-3)^k$$

Find the radius of convergence and the interval of convergence of the power series function *f*.

4. (5 pts.) Let $f(x) = \sqrt{x}$ *for* $x \geq 0$.

Obtain the 3rd Taylor polynomial of $f(x)$ about $x_0 = 1$.

$p_3(x) =$

5. (10 pts.) Suppose

$$f(x) = \sum_{k=1}^{\infty} \frac{(-1)^k}{k\,10^k}(x-2)^k$$

(a) By using sigma notation and term-by-term differentiation *as done in class*, obtain a power series for *f'*(*x*).

$f'(x) =$

(b) By using sigma notation and integrating term-by-term *as done in class*, obtain an infinite series whose sum has the same numerical value as that of the following definite integral. [We are working with the power series *f* above.]

$\int_2^3 f(x)\, dx =$

6. (10 pts.) (a) (2 pts.) Using literal constants A, B, C, etc., write the form of the partial fraction decomposition for the proper fraction below. Do not attempt to obtain the actual numerical values of the constants A, B, C, etc.

$\frac{3x^2+5}{x(x^2-1)(4x^2+1)^2} =$

(b) (3 pts.) Obtain the numerical values of the literal constants *A*, *B*, and *C* in the partial fraction decomposition given below.

$$\frac{2x^2+3x-1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

(c) (5 pts.) If one were to integrate the rational function in part (a), one probably would encounter the integral below. Find this integral.

$\int \frac{1}{4x^2+1}\, dx =$

7. (20 pts.) (a) Using a complete sentence, state the first part of the Fundamental Theorem of Calculus, the *evaluation* theorem.

(b) Using complete sentences, state the second part of the Fundamental Theorem of Calculus.

(c) Compute $g'(x)$ when $g(x)$ is defined by the following equation.

$$g(x) = \int_0^x \frac{1}{\sqrt{1 + t^2}}\, dt + \tan^{-1}(x)$$

$g'(x) =$

(d) Give the definition of the function $\ln(x)$ in terms of a definite integral, and give its domain and range. Label correctly.

(e) Write the solution to the following initial value problem in terms of a definite integral taken with respect to the variable *t*, so the differential denoting the variable of integration is *dt*. Then reveal an alternative identity of *y* by actually evaluating the definite integral with respect to *t*.

$$\frac{dy}{dx} = \frac{4\ln^3(x)}{x} \quad \text{with} \quad y(e) = 26.$$

$y(x) =$

8. (10 pts.) Find the area under the curve

$$y = \sqrt{16 - x^2}$$

from x = 0 to x = 2. First write the definite integral. Then evaluate it.

Area =

9. (8 pts.) (a) $\frac{\pi}{4} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2k-1}$

Use the error estimate from alternating series test to determine a specific value of $n \geq 1$ so that the partial sum s_n approximates $\pi/4$ to 5 decimal places, where, of course,

$$s_n = \sum_{k=1}^{n} \frac{(-1)^{k+1}}{2k-1}$$

(b) Use the *Remainder Estimation Theorem* to obtain an upper bound on the error of the following approximation when $0 \leq x \leq 0.1$.

$$\exp(x) \approx 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$$

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10. (6 pts.) Classify each of the following series as absolutely convergent (AC), conditionally convergent (CC), divergent (D), or none of the preceding, (N). Circle the letters corresponding to your choice. (No explicit proof is needed.)

(a) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^{1/4}+1}$ (AC) (CC) (D) (N)

(b) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k^{3/2}}{k+1}$ (AC) (CC) (D) (N)

(c) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!}$ (AC) (CC) (D) (N)

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11. (6 pts.) Here are three convergent infinite series that should be very easy to sum up at this stage. Provide the value of each sum.

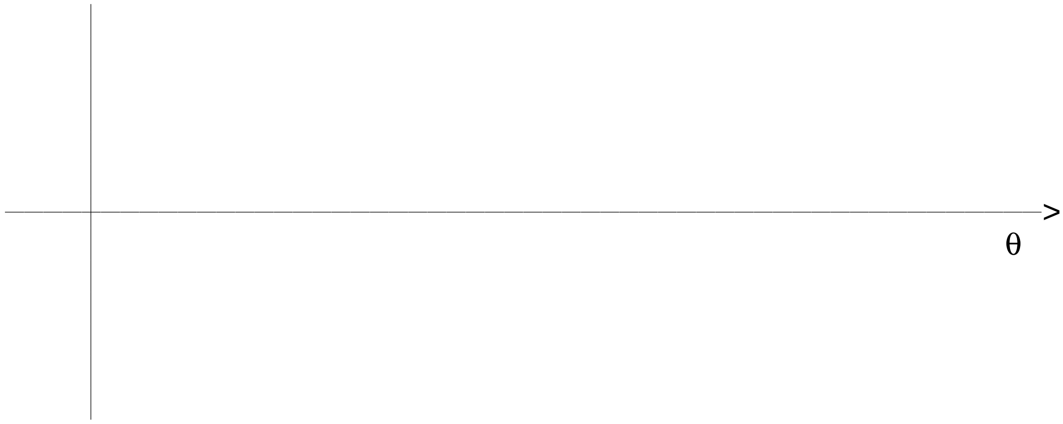
(a) $\sum_{k=0}^{\infty} \frac{(-1)^k (\pi/6)^{2k+1}}{(2k+1)!}$ =

(b) $\sum_{k=0}^{\infty} \left(\frac{1}{6}\right)\left(\frac{1}{2}\right)^k$ =

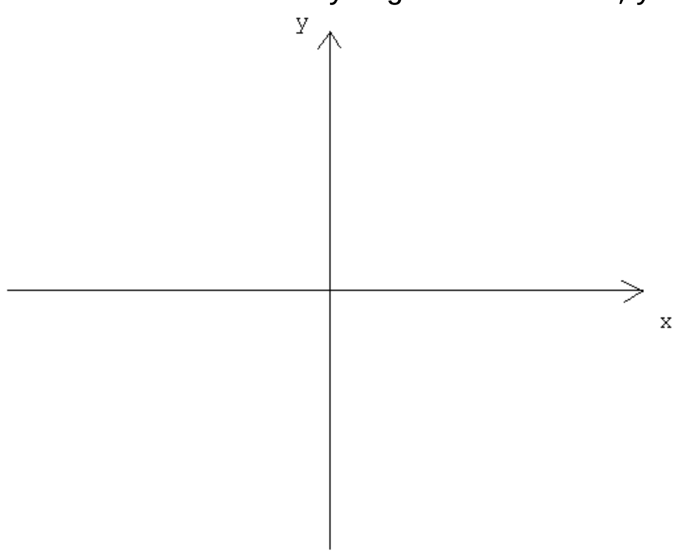
(c) $\sum_{k=4}^{\infty} \left(\frac{1}{k} - \frac{1}{k+1}\right)$ =

12. (6 pts.) Sketch the curve $r = \sin(2\theta)$ in polar coordinates.
Do this as follows: (a) Carefully sketch the auxiliary curve, a rectangular graph, on the r,θ-coordinate system provided. (b) Then translate this graph to the polar one.

(a) r ^



(b) [Think of polar coordinates lying over the x,y axes below.]



(c) Write down, but do not attempt to evaluate a definite integral that provides the numerical value of the area of one of the rose petals above.

Area =

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13. (4 pts.) Evaluate the following integral:

$$\int_0^\infty 2xe^{-x^2}\,dx =$$

Silly 10 point bonus: You may do at most one of the following 3 bonus problems. Clearly indicate which one and where your work is. (A) State and prove integral test.
(B) Let $f(x)$ be the power series function of Problem 5 on Page 3 of 6. Obtain a closed form for the numerical infinite series given by $f(3)$. In short, identify the sum of the series. (C) State the Mean-Value Theorem for Integrals and use it to prove the second part of the Fundamental Theorem of Calculus.