**READ ME FIRST:** *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⟹" denotes "implies" , and "⟺" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.*

1. (25 pts.)   The region ~~in the first quadrant~~ enclosed by the curves $y = 1 - x$, $y = \ln x$ and $x = e$ is sketched below for your convenience. [*Corrected in Class.*]
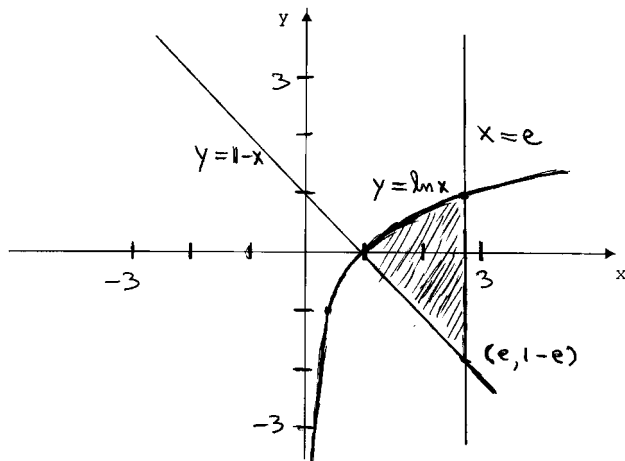
(a) Write down, but do not attempt to evaluate the definite integral whose numerical value gives the area of the region R if one integrates with respect to x so the differential in the integral is dx.

  Area =

(b) Write down, but do not attempt to evaluate the sum of definite integrals whose numerical value gives the area of the region R if one integrates with respect to y so the differential in the integral is dy.

  Area =

(c) Using the method of cylindrical shells, write a single definite integral dx whose numerical value is the volume of the solid obtained when the region R above is revolved around the y-axis. Do not evaluate the integral.

  Volume =

(d) Using the method of disks or washers, write down a sum of definite integrals dy to compute the same volume as in part (c). Do not evaluate the integrals.

  Volume =

(e) Write down, but do not attempt to evaluate, the definite integral that gives the arc-length of the curve $y = \ln(x)$ from $x = 1$ to $x = e$.

  Length =

2. (15 pts.) (a) (10 pts.) Using literal constants A, B, C, etc., write the form of the partial fraction decomposition for the proper fraction below. Do not attempt to obtain the actual numerical values of the constants A, B, C, etc. Be very careful here.

$$\frac{4x^2+5}{x(x+1)^2(4x^2+1)^2} =$$

(b) (5 pts.) If one were to integrate the rational function in part (a), one probably would encounter the integral below. Reveal, in detail, how to evaluate this integral.

$$\int \frac{1}{(4x^2+1)^2} \, dx =$$

3. (60 pts.)   Evaluate each of the following antiderivatives or definite integrals. Give exact values for definite integrals.
[**6 pts./part**]

(a) $\int 4x\cos(2x) \, dx =$

(b) $\int_0^{(\pi/6)^{1/2}} 4x\sin(x^2) \, dx =$

3. (Continued) Evaluate each of the following antiderivatives or definite integrals. Give exact values for definite integrals.
[**6 pts./part**]

(c) $\int \sin^{-1}(x)\ dx =$

(d) $\int 8x^2 e^x dx =$

(e) $\int \dfrac{2}{\sin(t)\cos(t)}\ dt =$

(f) $\int \sqrt{4 - x^2}\ dx =$

3. (Continued) Evaluate each of the following antiderivatives or definite integrals. Give exact values for definite integrals.

(g) $\int \dfrac{2x^4 + 4x^3}{x^2 + 1}\ dx =$

(h) $\int \sec(4t)\ dt =$

(i) $\int \dfrac{12}{x^2 - 9}\ dx =$

(j) $\int \cos(x)\,e^x\ dx =$

Silly 10 Point Bonus: State and prove the Mean-Value Theorem of Integrals. In doing this, feel free to use the Extreme-Value Theorem and the Intermediate-Value Theorems, which codify important behaviors of continuous functions. //Say where your work is, for there isn't room here.