
READ ME FIRST: Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals", " $\Rightarrow$ " denotes "implies", and " $\Leftrightarrow$ " denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page. Eschew obfuscation.

1. (10 pts.) Consider the definite integral below. (a) Write down the sum, S_6 , used to approximate the value of the integral below if Simpson's Rule is used with $n = 6$. Do not attempt to evaluate the sum. (b) Write down the sum, T_6 , used to approximate the value of the integral below if Trapezoid Rule is used with $n = 6$. Do not attempt to evaluate the sum.

$$\int_0^3 \sqrt{x} \, dx$$

(a) $S_6 =$

(b) $T_6 =$

2. (10 pts.) Evaluate the integrals that converge.

(a) $\int_{-\infty}^3 \frac{2x \, dx}{x^2 + 9} =$

(b) $\int_{\frac{1}{2}}^1 \frac{2}{\sqrt{1-x^2}} \, dx =$

3. (4 pts.) Express the repeating decimal as a fraction, more specifically as a quotient of positive integers. [The fraction does not have to be in lowest terms.]

$$0.75757575 \dots =$$

4. (4 pts.) Find the general term of the sequence, starting with $n = 1$, determine whether the sequence converges, and if so, find its limit.

$$\frac{1}{3}, -\frac{2}{5}, \frac{3}{7}, -\frac{4}{9} \dots$$

5. (4 pts.) [Complete the following.] The harmonic series has the form

$$\sum_{k=1}^{\infty}$$

What is the sum of the harmonic series?

6. (8 pts.) Determine whether the series converges, and if so, find its sum.

(a) $\sum_{k=1}^{\infty} \left(\frac{2}{3} \right)^{k+2}$

(b) $\sum_{k=1}^{\infty} \left(\frac{10}{k+4} - \frac{10}{k+5} \right)$

7. (4 pts.) Use root test to determine whether the series converges. If the test is inconclusive, say so.

$$\sum_{k=1}^{\infty} \left(\frac{3k+2}{2k-1} \right)^k$$

8. (4 pts.) Apply the divergence test and state what it tells you about each of the following series.

(a) $\sum_{k=1}^{\infty} \left(1 + \frac{1}{k} \right)^k$

(b) $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$

9. (4 pts.) Use ratio test to determine whether the series converges. If the test is inconclusive, say so.

$$\sum_{k=1}^{\infty} \frac{1}{k^2}$$

10. (4 pts.) Use comparison test to show the following series diverges.

$$\sum_{k=1}^{\infty} \frac{\ln(k)}{k}$$

11. (4 pts.) Confirm that the integral test is applicable, and then use it to determine whether the following series converges:

$$\sum_{k=1}^{\infty} \frac{1}{1+k^2}$$

12. (4 pts.) Find all values of x for which the series converges, and find the sum of the series for those values of x .

$$\frac{1}{x^2} + \frac{2}{x^3} + \frac{4}{x^4} + \frac{8}{x^5} + \frac{16}{x^6} + \dots$$

13. (6 pts.) [Complete the following.] A geometric series is a series of the form

$$\sum_{k=0}^{\infty}$$

This series diverges if _____ and this series converges if _____.

14. (4 pts.) Suppose that

$$\sum_{n=1}^{\infty} a_n = \frac{2}{5}$$

and

$$\sum_{n=1}^{\infty} b_n = -6.$$

Then we have

$$\sum_{n=1}^{\infty} \left(60a_n + \frac{5}{3}b_n \right) =$$

15. (6 pts.) [Complete the following.] A p-series is a series of the form

$$\sum_{k=1}^{\infty}$$

This series diverges if _____ and this series converges if _____.

16. (8 pts.) (a) Using complete sentences and appropriate notation, give the precise ε - N definition of

$$\lim_{n \rightarrow \infty} a_n = L.$$

(b) Give the precise mathematical definition of the sum of an infinite series,

$$\sum_{k=1}^{\infty} a_k$$

17. (4 pts.) From the definition of a limit of sequence, we know there is a positive integer N so that if $n \geq N$, then

$$\left| \frac{n}{n+1} - 1 \right| < \left(\frac{1}{2} \right) 10^{-3}$$

since

$$\lim_{n \rightarrow \infty} \frac{n}{n+1} = 1.$$

Find a positive integer N which works and prove it provides the desired error bound. [You don't have to get the best. Just obtain one the you can prove works, and then convince the skeptics.]

18. (8 pts.) Let the sequence $\{a_n\}$ be defined recursively by

$$a_1 = \sqrt{2}, \text{ and } a_{n+1} = \sqrt{2 + a_n}$$

for $n \geq 1$. (a) List the first four terms of the sequence.

(b) Assuming the sequence converges, find its limit L .

Silly 10 Point Bonus: Prove that

$$\sum_{k=1}^n \frac{1}{k^4} < \frac{4}{3}$$

for every $n \geq 1$. [Say where your work is, for it won't fit here.]