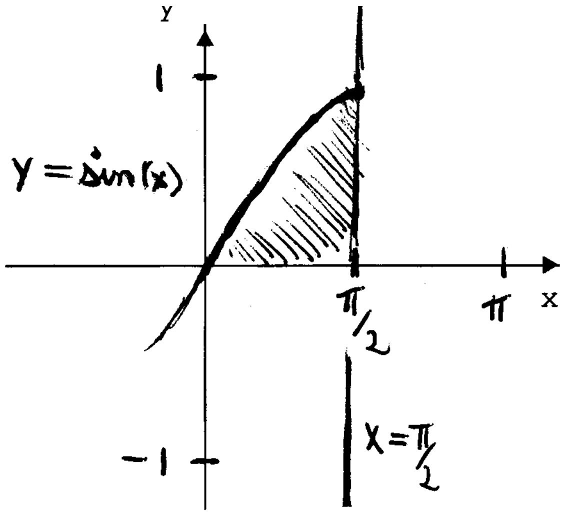
READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Eschew obfuscation. Show me all the magic on the page.*

1. (25 pts.) The region R in the first quadrant enclosed by the curves defined by $y = \sin x$, $y = 0$, and $x = \pi/2$ is sketched below for your convenience.



(a) Write down, but do not attempt to evaluate the definite integral whose numerical value gives the area of the region R if one integrates with respect to *x* so the differential in the integral is *dx*.

Area =

(b) Write down, but do not attempt to evaluate the definite integral whose numerical value gives the area of the region R if one integrates with respect to *y* so the differential in the integral is *dy*.

Area =

(c) Using the method of cylindrical shells, write a single definite integral *dy* whose numerical value is the volume of the solid obtained when the region R above is revolved around the x-axis. Do not evaluate the integral.

Volume =

(d) Using the method of disks or washers, write down a definite integral *dx* to compute the same volume as in part (c). Do not evaluate the integrals.

Volume =

(e) Write down, but do not attempt to evaluate, the definite integral that gives the arc-length of the curve $y = \ln(x)$ from $x = 1$ to $x = e$.

Length =

2. (15 pts.) (a) (5 pts.) Using literal constants A, B, C, etc., write the form of the partial fraction decomposition for the proper fraction below. Do not attempt to obtain the actual numerical values of the constants A, B, C, etc. Be very careful here.

$$\frac{4x^2+5}{x(3x-5)^3(9x^2+1)^2} =$$

(b) (5 pts.) If one were to integrate the rational function in part (a), one might encounter the integral below. Evaluate the integral below.

$$\int \frac{1}{(3x-5)^3}\,dx =$$

(c) (5 pts.) If one were to integrate the rational function in part (a), one might also encounter the integral below. Evaluate the integral below.

$$\int \frac{x}{(9x^2+1)^2}\,dx =$$

3. (60 pts.) Evaluate each of the following antiderivatives or definite integrals. Give exact values for definite integrals.
[**6 pts./part**]

(a) $\int_0^{\sqrt{\pi/3}} 6x\cos(x^2)\,dx =$

(b) $\int 6x\sin(2x)\,dx =$

3. (Continued) Evaluate each of the following antiderivatives or definite integrals. Give exact values for definite integrals.
[**6 pts./part**]

(c) $\int 16x^2e^{4x}\,dx =$

(d) $\int_0^1 \tan^{-1}(x)\,dx =$

(e) $\int \frac{2}{\sqrt{1 + 4t^2}}\,dt =$

(f) $\int \frac{x^2}{\sqrt{1 - x^2}}\,dx =$

3. (Continued) Evaluate each of the following antiderivatives or definite integrals. Give exact values for definite integrals.

(g) $\int \frac{x^4 - 5x^3}{x^2 + 1}\, dx =$

(h) $\int 4\cos(4t)\cos(2t)\, dt =$

(i) $\int \frac{50}{\theta^2 - 25}\, d\theta =$

(j) $\int \sec^3(x)\, dx =$

Silly 10 Point Bonus: (a) Prove that

$$(*) \qquad \frac{1}{t} \geq 1 - \frac{t}{4}$$

for every t ε [1,3]. (b) Using (a) to compare a couple of integrals, prove ln(3) > 1. (c) Using part (b) and the Intermediate-Value Theorem for Continuous Functions, prove that there is a unique number x_0 in the open interval (2,3) where ln(x) = 1. //Say where your work is, for there isn't room here.