READ ME FIRST: Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page. Eschew obfuscation.

1. (10 pts.) Consider the definite integral below. (a) Write down the sum, S_4, used to approximate the value of the integral below if Simpson's Rule is used with $n = 4$. Do not attempt to evaluate the sum. (b) Write down the sum, T_4, used to approximate the value of the integral below if Trapezoid Rule is used with $n = 4$. Do not attempt to evaluate the sum.

$$\int_2^4 \sqrt{x}\ dx$$

(a) $S_4 =$

(b) $T_4 =$

2. (10 pts.) Evaluate the integrals that converge.

(a) $\displaystyle\int_{-1}^{+\infty} \frac{2\,dx}{x^2 + 1} =$

(b) $\displaystyle\int_0^{\pi/2} \tan(x)\ dx =$

3. (4 pts.) Express the repeating decimal as a fraction, more specifically as a quotient of positive integers. [The fraction does not have to be in lowest terms.]

$0.54545454\ldots =$

4. (4 pts.) Find the general term of the sequence, starting with *n* = 1, determine whether the sequence converges, and if so, find its limit.

$$0, \frac{1}{2^3}, \frac{2}{3^3}, \frac{3}{4^3}, \frac{4}{5^3} \ldots.$$

5. (4 pts.) $\frac{\pi}{4} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2k-1}$

Use the error estimate from alternating series test to determine a specific value of $n \geq 1$ so that the partial sum s_n approximates $\pi/4$ to 5 decimal places, where, of course,

$$s_n = \sum_{k=1}^{n} \frac{(-1)^{k+1}}{2k-1}$$

6. (8 pts.) Determine whether the series converges, and if so, find its sum.

(*a*) $\displaystyle\sum_{k=1}^{\infty} \left(-\frac{2}{3}\right)^{k+2}$

(*b*) $\displaystyle\sum_{k=1}^{\infty} [\ln(k+2) - \ln(k+3)]$

7. (4 pts.) Use root test to determine whether the series converges. If the test is inconclusive, say so.

$$\sum_{k=1}^{\infty}\left(1+\frac{2}{k}\right)^{k}$$

8. (4 pts.) Apply the divergence test and state what it tells you about each of the following series.

(a) $\displaystyle\sum_{k=1}^{\infty}\frac{1}{k!}$

(b) $\displaystyle\sum_{k=1}^{\infty}\frac{\sqrt{k}}{\sqrt{k}+3}$

9. (4 pts.) Use ratio test to determine whether the series converges. If the test is inconclusive, say so.

$$\sum_{k=1}^{\infty}\frac{k}{5^{k}}$$

10. (4 pts.) Use comparison test to show the following series converges.

$$\sum_{k=1}^{\infty}\frac{\sqrt{k}}{k^{2}+4}$$

11. (4 pts.) Confirm that the integral test is applicable, and then use it to determine whether the following series converges:

$$\sum_{k=2}^{\infty}\frac{1}{k\ln^{2}(k)}$$

12. (4 pts.) Find all values of x for which the series converges, and find the sum of the series for those values of x.

$$\frac{1}{x^2} + \frac{5}{x^3} + \frac{25}{x^4} + \frac{125}{x^5} + \frac{625}{x^6} + \ldots$$

13. (6 pts.) Classify each of the following series as absolutely convergent (AC), conditionally convergent (CC), divergent (D), or none of the preceding, (N). Circle the letters corresponding to your choice. (No explicit proof is needed.)

(a) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^{1/4}+1}$ (AC) (CC) (D) (N)

(b) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k^{3/2}}{k+1}$ (AC) (CC) (D) (N)

(c) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!}$ (AC) (CC) (D) (N).

14. (4 pts.) The following series diverges:

$$(*) \quad \sum_{k=1}^{\infty} \left[\frac{1}{3k+2} - \frac{1}{k^{3/2}} \right]$$

Provide the indirect reasoning that gives a proof of this fact.

15. (6 pts.) [Complete the following.] A p-series is a series of the form

$$\sum_{k=1}^{\infty}$$

This series diverges if _______________ and this series converges if

______________.

16. (8 pts.) (a) Using complete sentences and appropriate notation, give the precise ε - N definition of

$$\lim_{n \to \infty} a_n = L.$$

(b) Give the precise mathematical definition of the sum of an infinite series,

$$\sum_{k=1}^{\infty} a_k$$

17. (4 pts.) From the definition of a limit of sequence, we know there is a positive integer N so that if $n \geq N$, then

$$\left| \frac{n}{2n+2} - \frac{1}{2} \right| < \left(\frac{1}{2} \right) 10^{-3}$$

since

$$\lim_{n \to \infty} \frac{n}{2n+2} = \frac{1}{2}.$$

Find a positive integer N which works and prove it provides the desired error bound.

18. (8 pts.) Let the sequence $\{a_n\}$ be defined recursively by

$$a_1 = \sqrt{3}, \quad and \quad a_{n+1} = \sqrt{3 + a_n}$$

for $n \geq 1$. (a) List the first four terms of the sequence.

(b) Assuming the sequence converges, find its limit L.

Silly 10 Point Bonus: Prove that for every positive integer $n \geq 1$,

$$\sqrt{n+1} - 1 < \sum_{k=1}^{n} \frac{1}{2\sqrt{k}}$$

[Say where your work is, for it won't fit here.]