NAME:

STUDENT NUMBER: **EXAM NUMBER:**

READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Since the answer really consists of all the magic transformations and incantations, do not "box" your final results. Show me all the magic on the page.*

1. (a)(10 pts.) Find both the Maclaurin polynomial, $P_2(x)$, and the Lagrange form of the remainder term, $R_2(x)$, for the function $f(x) = (1 + x)^{1/2}$. Then write $(1 + x)^{1/2}$ in terms of $P_2(x)$ and $R_2(x)$.

$P_2(x) =$

$R_2(x) =$

$(1 + x)^{1/2} =$

(b) (5 pts.) Using part (a), provide a reasonable estimate of how many decimal places accuracy your computation would provide if you used $P_2(x)$ to approximate $(1 + x)^{1/2}$ with $|x| \le 0.1$.

2. (10 pts.) Using sigma notation, write the Maclaurin series of each of the following functions, and give the interval I where the series converges to the function.

$\cos(x) =$

$\ln(1+x) =$

$\tan^{-1}(x) =$

$\sin(x) =$

$e^x =$

3. (15 pts.) Suppose that

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n5^n}(x-5)^n$$

(a) (5 pts.) Find the radius of convergence and the interval of convergence of the power series function f.

(b) (5 pts.) By using sigma notation and doing termwise differentiation, obtain a power series for f′(x). What is the radius of convergence of the series for f′(x)?

f′(x) =

(c) (5 pts.) Obtain an infinite series whose sum is the same as the numerical value of the following definite integral. [We are working with the function f of part (a) of this problem. Integrate termwise. **Use sigma notation**.]

$$\int_5^6 f(x)dx =$$

4. (10 pts.) (a) Find the rational number represented by the following repeating decimal.

0.630630630... =

(b) Find all values of x for which the given geometric series converges, and then express the closed form sum of the series as a function of x.

$$\sum_{n=1}^{\infty}\left(\frac{(x-10)}{5}\right)^n =$$

5. (25 pts.) Find each of the following antiderivatives.

(a) $\int (1 - 16x^2)^{1/2}\, dx =$

(b) $\int \frac{x^3 - 2x}{x^2 + 1}\, dx =$

(c) $\int \cos(3x)\cos(2x)\, dx =$

(d) $\int \ln(x^2 + 1)\, dx =$

(e) $\int \frac{x + 1}{x(x^2 + 1)}\, dx =$

6. (25 pts.) (a) (7 pts.) State the Fundamental Theorem of Calculus.

(b) (6 pts.) Compute g′(x) when g(x) is defined by the following equation.

$$g(x) = \int_0^x \tan^3(t)\, dt + \sec^3(x)$$

g′(x) =

(c) (6 pts.) Write the solution to the following initial value problem in terms of a definite integral with respect to the variable t, but don't attempt to evaluate the definite integral:

$$y'(x) = e^{\sec(x)\tan(x)}, \quad y(0) = \pi/3.$$

y(x) =

(d) (6 pts.) Suppose that f(0) = 1/2 and

$$f'(x) = \frac{8x}{1 + 4x^2} .$$

What function f(x) satisfies these two equations?? [Identify f as completely as possible. f can be written in terms of an old friend.]

f(x) =

---

7. (25 pts.) Obtain the exact numerical value of each of the following if possible. If a limit doesn't exist or an improper integral or an infinite series fails to converge, say so as precisely as possible. [Warning: If you evaluate improper integrals improperly, you will lose most of the points possible on the problem part. Pay attention to details.]

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{\pi}{n^2 - n} =$

(b) $\displaystyle\int_1^{\infty} \frac{8}{x^2 + 1}\, dx =$

(c) $\displaystyle\int_2^{\infty} \frac{\pi}{x^2 - x}\, dx =$

(d) $\displaystyle\lim_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} \sec\left(\frac{\pi}{4}\left(\frac{k}{n}\right)\right) =$

(e) $\displaystyle\int_0^{1} \frac{x^2}{(1 - x^2)^{1/2}}\, dx =$

8. (10 pts.) (a) Since the third and fourth Maclaurin polynomials for sin(x) are the same, it follows from Taylor's Theorem that if x is a real number different from 0, we may write

$$\sin(x) = x - \frac{1}{6}x^3 + \frac{\cos(c)}{120}x^5,$$

where c is some number between x and 0. Using this equation, obtain an open interval that is centered at 0 where sin(x) may be approximated to 5 decimal place accuracy using the polynomial

$$P_4(x) = x - (1/6)x^3.$$

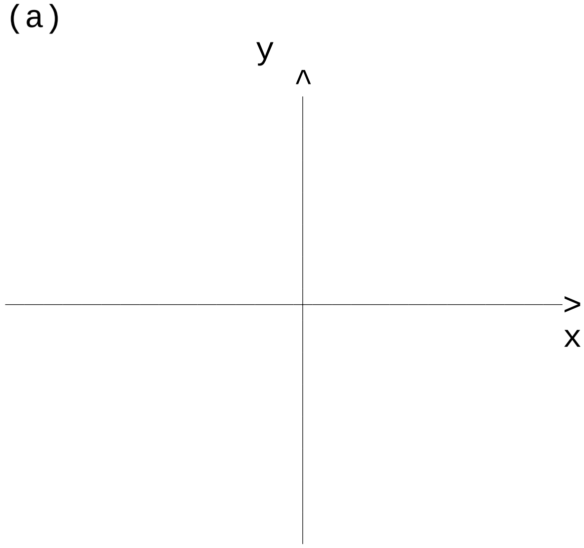
(b) What is the exact value of the sum of the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n(\pi/3)^{2n}}{(2n)!} \text{ ?}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n(\pi/3)^{2n}}{(2n)!} =$$

9. (15 pts.) (a) Sketch the region enclosed by the curves y = cos(x) and y = 0 between x = 0 and x = π/2. Label carefully. (b) If the region is revolved around the y-axis, a solid is formed. Using the method of disks and washers, write down a single definite integral with respect to the variable y which would be used to compute the volume of the solid. **Don't attempt to evaluate the integral**. (c) Using the method of cylindrical shells, write down the definite integral with respect to the variable x that would be used to compute the same volume of the solid of part (b). **Don't attempt to evaluate the integral.**

(a)



(b)

V =

(c)

V =

10. (16 pts) (a) (10 pts.) Using literal constants A, B, C, etc., write the form of the partial fraction decomposition for the proper fraction below. Do not attempt to obtain the actual numerical values of the constants A, B, C, etc.

$$\frac{x^2+25}{(x-2)(x^2+1)^2} =$$

(b) (6 pts.) Now obtain the indefinite integral of the rational function of part (a) in terms of the constants A, B, C, etc. using the partial fraction decomposition. Do not attempt to obtain the numerical values of the constants. A trig substitution somewhere along the way *might* help.

$$\int \frac{x^2+25}{(x-2)(x^2+1)^2}\, dx =$$

---

11. (9 pts.) Classify each of the following series as absolutely convergent, conditionally convergent, or divergent. (3 pts./part)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{3/2}+1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}+1}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+\sqrt{n}}$

12. (10 pts.) Evaluate completely exactly one of the following two definite integrals. Very clearly indicate which you intend to attempt by circling the appropriate letter.

(a) $\int_0^1 (1 + x^2)^{1/2}\, dx$

(b) $\int_0^\infty \cos(t)e^{-t}\, dt$

Show all essential details. These may include things like trigonometric substitutions and the use of the "squeeze" theorem in the evaluation of limits. [Don’t expect to do either of these "in your head".]

Silly 20 point bonus: (a) It turns out that if $\{a_n\colon n \geq 1\}$ is a convergent sequence, that is

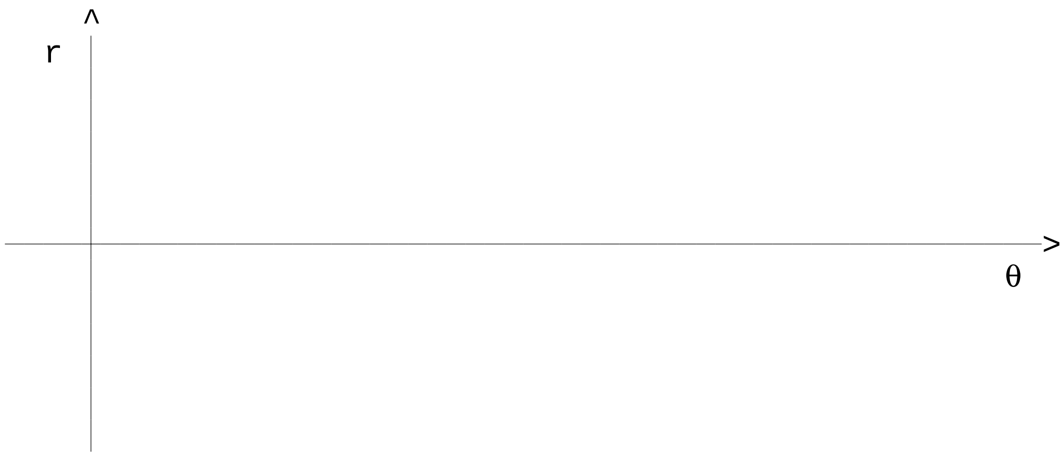
$$\lim_{n\to\infty} a_n = L$$

for some honest to goodness real number L, then the number L may be written easily as the sum of an infinite series that involves the use of the elements of the sequence $\{a_n\}$ in an essential way. Can you reveal how this is done, in detail, with proof?? Where is your work??

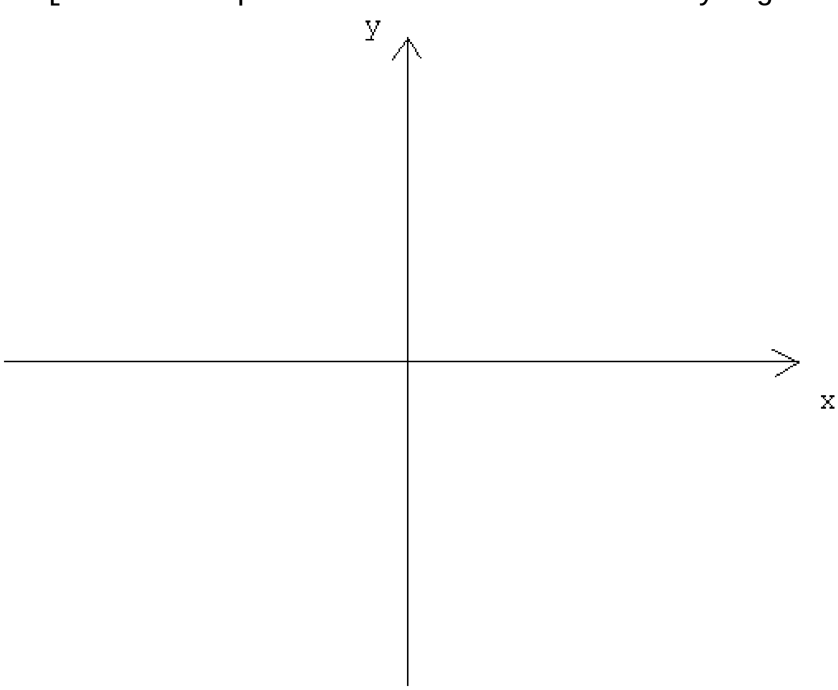
(b) Show how to obtain a rational number that approximates $\tan^{-1}(10)$ to 6 decimal places, with proof the process works. Where’s your work??

(15 pts.) Sketch the curve $r = 2\cdot\cos(2\theta)$ in polar coordinates, and then compute the area of the region enclosed by one loop of the curve. Do this as follows: (a) Carefully sketch the auxiliary curve, a rectangular graph, on the coordinate system provided. (b) Then translate this graph to the polar one. Finally, (c) setup the required polar integral and evaluate it. [Hint: Look at the graph in part (a) to positively choose a suitable loop.]

(a)



(b) [Think of polar coordinates overlaying the x,y axes below.]



(c)

Area =