READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magical transformations, do not "box" your final results. Communicate. Show me all the magic on the page, for I do not read minds.*

1. (10 pts.) (a) Write the value of the following limit as a definite integral.

$$L = \lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} \cos\left(\frac{\pi}{2}\left(\frac{k}{n}\right)\right).$$

L =

(b) By evaluating the integral you obtained in part (a) above using the Fundamental Theorem of Calculus, give the exact numerical value of the limit, L.

L =

2. (10 pts.) Evaluate each of the following sums in closed form.

(a) $\displaystyle\sum_{i=0}^{199}\left(\frac{1}{5}\right)^{i} =$

(b) $\displaystyle\sum_{i=1}^{200}(4i - 1) =$

3. (10 pts.) (a) State the Fundamental Theorem of Calculus.

(b) Write the solution to the following initial value problem in terms of a definite integral with respect to the variable t, but don't attempt to evaluate the definite integral:

$$y'(x) = e^{\tan(x)}, \quad y(\pi/3) = 8.$$

y(x) =

4. (10 pts.) Reveal all the details of evaluating the given integral by computing

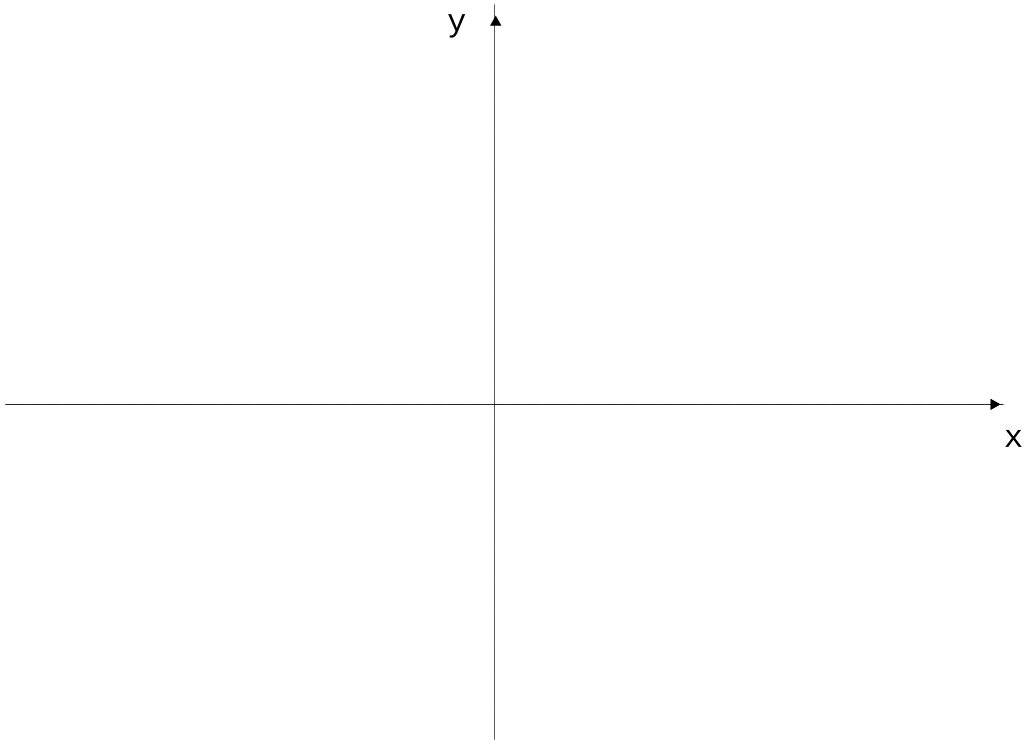
$$\lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

where the sum is assumed to have originated from a regular partition of the given interval of integration. Here, you are to actually compute the Riemann sum in closed form and then evaluate the limit. Suppose that $b > 0$ below. Then

$$\int_0^b x^2\, dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x =$$

5. (15 pts.) (a) Sketch very carefully the bounded region bounded by the curves $y = 1 - x$, $y = \ln x$ and $x = e$ on the coordinate system provided. Label very carefully. (b) Write down a single integral, dx, that provides the numerical value of the area of the region. (c) Write a sum of definite integrals, dy, that yields the area of the region. Do not attempt to evaluate the definite integrals.

(a)



(b) Area =

(c) Area =

6. (5 pts.) Suppose that g is defined on the interval $[\pi, 4\pi]$ by means of the equation

$$g(x) = \int_{2\pi}^{x} \frac{\cos(t)}{t}\, dt.$$

Determine the open intervals in $(\pi, 4\pi)$ where g is increasing or decreasing.

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7. (10 pts.) Differentiate the following functions:
[Make sure you label your derivatives correctly.]

(a) $g(x) = \int_0^x \sec^3(t)\,dt + \tan(x)$, *for* $x \in (-\pi/2, \pi/2)$.

(b) $f(x) = \int_0^{\ln(x)} \frac{1}{1+t^2}\,dt$, *for* $x > 0$.

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8. (10 pts.) (a) Using only the second comparison property of integrals, give both a lower bound and an upper bound on the true numerical value of the integral below.

$$I = \int_1^2 \frac{1}{t}\,dt.$$

What does this tell you about ln(2)?

(b) Using appropriate properties of the definite integral and suitable area formulas from geometry, evaluate the following definite integral. [Roughly sketching a couple of simple graphs might help. First use linearity, though.]

$\int_{-2}^{0} x + (4-x^2)^{1/2}\,dx =$

9. (10 pts.)
(a) Find the average value of $f(x) = \sec^2(x)$ over the interval $[\pi/4, \pi/3]$.

f_{AVE} =

(b) What must the constants *a* and *b* be for the following equation to be valid when we use the substitution u = tan(x)??

$$\int_{-\pi/3}^{\pi/4} \tan^2(x)\sec^2(x)\,dx = \int_a^b u^2\,du.$$

a =

b =

10. (10 pts.) Consider the definite integral below. (a) Write down the sum, S_4, used to approximate the value of the integral below if Simpson's Rule is used with n = 4. **Do not attempt to evaluate the sum. Be very careful.**

$$\int_4^6 x^{1/2}\,dx$$

(a) S_4 =

(b) The magnitude of error in using Simpson's Rule to approximate the definite integral of f(x) on an interval [a,b] with *n* subintervals may be estimated using

$$|ES_n| \le \frac{M_4(b-a)^5}{180n^4},$$

provided the fourth derivative of f, $f^{(4)}(x)$, is continuous on [a,b], and $|f^{(4)}(x)| \le M_4$ for each x in [a,b].

Use this information to determine how big *n* must be to ensure that you will obtain an approximation to the true value of the integral above that is accurate to 6 decimal places when using Simpson's Rule.

Silly 10 Point Bonus: Provide a complete proof of the first part of the Fundamental Theorem of Calculus as given by Edwards and Penney. Say where your work is, for it won't fit here.