READ ME FIRST: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page, for I do not read minds. Eschew obfuscation.*

1. (5 pts.) Express the following sum using sigma notation, but do not attempt to find its numerical value.

$$(2^3+3^4)-(3^3+3^5)+(4^3+3^6)-(5^3+3^7)+(6^3+3^8)-(7^3+3^9) =$$

2. (5 pts.) Obtain the upper limit of summation and rewrite the summand in order to make the following equation true. **Do not attempt to evaluate the sum.**

$$\sum_{k=9}^{55} 10^{2k} = \sum_{j=1}$$

3. (5 pts.) If the function f is continuous on [a,b], then the *net signed area* A between $y = f(x)$ and the interval [a,b] is defined by

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x.$$

Reveal all the details in obtaining the numerical value of the net signed area of $f(x) = 2x$ over the interval [0,4] using the definition above with

$$x_k^*$$

the right end point of each subinterval in the regular partition.

4. (5 pts.) Express the following limit as a definite integral. Do not attempt to evaluate the integral.

$$L = \lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} 4x_k^*(1 - 3x_k^*)\Delta x_k \ ; \quad a = -3,\ b = 4.$$

L =

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5. (10 pts.) Suppose that f(x) is an integrable function such that

$$\int_{-2}^{1} f(x)\,dx = 4 \quad \textit{and} \quad \int_{1}^{2} f(x)\,dx = -2.$$

Use this information and appropriate formulas from geometry to evaluate the following definite integrals:

(a) $\int_{1}^{2} 6f(x)\,dx =$

(b) $\int_{-2}^{1} 4 - 2f(x)\,dx =$

(c) $\int_{-2}^{2} f(x)\,dx =$

(d) $\int_{-2}^{2} f(x) + (4 - x^2)^{1/2}\,dx =$

(d) $\int_{2}^{1} f(x)\,dx =$

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6. (5 pts.) Using a complete sentence and appropriate notation, state precisely the First Part of the Fundamental Theorem of Calculus. [This is sometimes called the *Evaluation Theorem*.]

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7. (5 pts.) Find the area under the curve

$$y = \frac{1}{(x+4)^2}$$

over the interval [-2,1].

Area =

8. (5 pts.) If the given integral is expressed as an equivalent integral in terms of the variable u using the substitution $u = x^3 - 4$ so that the equation below is true, what are the numerical values of the new limits of integration α and β, and what is the new integrand, $f(u)$??? Obtain these but do not attempt to evaluate the du integral.

$$\int_{-1}^{2} \pi x^2 (x^3 - 4)^5 \, dx = \int_{\alpha}^{\beta} f(u) \, du$$

α =

β =

$f(u)$ =

9. (5 pts.) Using complete sentences and appropriate notation, state the Second Part of the Fundamental Theorem of Calculus.

10. (10 pts.) Let the function g be defined by the equation

$$g(x) = \int_{3^{1/2}}^{x} \tan^{-1}(t) \, dt$$

for $x \in (-\infty, \infty)$. Then

(a) $g(\sqrt{3})$ =

(b) $g'(\sqrt{3})$ =

(c) $g''(\sqrt{3})$ =

(d) Determine the open intervals where g is increasing or decreasing. Be specific.

(d) Determine the open intervals where g is concave up or concave down. Be specific.

11. (5 pts.) Write the solution to the following initial value problem in terms of a definite integral taken with respect to the variable **t**, so the differential denoting the variable of integration is **dt**. Do not attempt to evaluate the definite integral you get.

$$\frac{dy}{dx} = \sec^3(x) \text{ with } y(-\pi/4) = 2$$

$y(x) =$

12. (10 pts.) A particle moves with a velocity of $v(t) = \cos(t)$ along an s-axis. Find the displacement and total distance traveled over the time interval $[\pi/2, 2\pi]$.

Displacement =

Total_Distance =

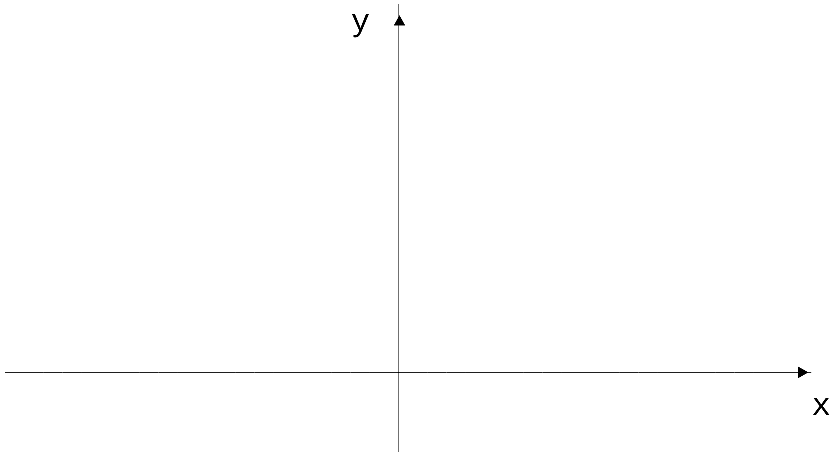
13. (5 pts.) (a) (3 pts.) Give the definition of the function ln(x) in terms of a definite integral and give its domain and range. Label correctly. (**Hint**: Complete the sentence,"ln(x) = ...")

(b) (2 pts.) Given ln(a) = 4 and ln(c) = 5, evaluate the following integral.

$$\int_c^{a^4} \frac{1}{\omega}\, d\omega =$$

14. (10 pts.) (a) Sketch the region enclosed by the curves defined by $y = x^2$ and $y = x + 2$. (b) Then give a single definite integral dx whose numerical value is the area of the region. (c) Finally, evaluate the integral of part (b).

(a)



(b)

15. (5 pts.) Express the function $g(x)$ below in a piecewise defined form that does not involve an integral.

$$g(x) = \int_0^x |t| \, dt$$

16. (5 pts.) Evaluate the following limit:

$$L = \lim_{x \to \infty} \left(1 + \frac{3}{x}\right)^{2x} =$$

Silly 10 Point Bonus: (a) Prove that the function F(x) defined by the equation

$$F(x) = \int_0^x \frac{1}{1+t^2} \, dt + \int_0^{1/x} \frac{1}{1+t^2} \, dt$$

is constant on the interval $(0,\infty)$. (b) Using (a), prove that for $x > 1$ that

$$\int_1^x \frac{1}{1+t^2} \, dt < \int_0^1 \frac{1}{1+t^2} \, dt.$$